GI ROC Reserving Study - 2009

David Munroe (Insureware), David Odell (Insureware), Ben Zehnwirth (Insureware)

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Introduction and Summary

To each of the datasets D, F, H and I we identify (design or build) the optimal model in the Probabilistic Trend Family (PTF) modelling framework for the incremental paid losses and the Case Reserve Estimates (CREs).

A model belonging to the PTF modelling framework is depicted by four graphs; the trend structure for each of the three directions development period, accident period and calendar period, and the quality of the process variance about the trend structure. Forecasting scenarios for the paid losses are based on the information extracted from the paid losses in respect of stability of calendar year trends, and any identified 'trend relationship' between the paid losses and the CREs. In the PTF modelling framework the actuary has control on formulating forecast scenarios for the future related to past experience. These scenarios are explicit, audit able and can be monitored in a sound probabilistic framework.

The Mack Method is also applied to the cumulative paid losses and the incurred losses. These methods are tested in respect of capturing the volatility in the data and also in respect of their degree of predictive power.

In order to calculate the Cost of Capital the probability distribution by calendar of the liability stream needs to be computed. We do this based on explicit and auditable assumptions that can be monitored on updating in a sound probabilistic framework. The volatility in the future paid losses cannot be extracted from the incurred losses array, notwithstanding the fact that for most cumulative arrays the Mack and related link ratio methods give grossly inaccurate indications.

Summary of results

In general, standardized residuals of a fitted model exhibit the remaining structure in the data adjusted for the fitted parameters. Equivalently, they represent trends in the data minus the trends estimated by the model. For an optimal model residuals are random around zero so that trends in the data equal the trends estimated by the model.

Dataset D

The identified model structure for dataset D did not have any calendar year trend changes in the available data. As a result, the modelling and choice of future structure was straightforward. Monitoring will be important in order to pick up on any trend changes should they occur in the future.

The Mack method, when applied to this dataset, while it gave answers which were comparable to the optimal PTF model (though on the low side), does not have predictive power, does not quantify the structure in the data and therefore is not preferable for selection.

Furthermore, the Mack method does not project volatility in the liability stream – a necessity for any cost of capital allocation.

Dataset F

In the paid losses to date, no calendar trend changes have been observed. There have been two increases by accident period, however, and these would need to be taken into consideration when computing any underwriting risk. By contrast, the Case Reserve Estimates (CREs) have an identified trend of 22%+_3.5% for the most recent two years. It is necessary to determine the reason for this increase before applying this increase to the Paid Losses.

Once again, with no calendar trend changes (as found in the optimal PTF model), the Mack method still suffered from the same problems as observed for dataset D – under projection of the total reserve and the ratios lacking in predictive power.

The optimal PTF model was selected.

Dataset H

Both the (incremental) paid losses and the case reserve estimates (CREs) possess major trend shifts in recent calendar years. These suggest shifts in closure rates. The trend change in the Paid Losses is in the opposite direction as compared to the CRE. The relationship between the two data types is suggestive of a hypothesised forecasting scenario going forward. This hypothesised scenario could be more fully tested if we had access to the number of closed claims (NCC) triangle.

For this dataset the Mack Method applied to the cumulative paid losses and the (cumulative) incurred losses gives answers that are ridiculously low. In order to obtain the same mean given by Mack for the Incurred Losses from the optimal PTF model, we need to assume a calendar year trend of -25% +_ 3.46% over a (future) 10 year period. Another PTF scenario which gives the same answer as the Mack method on Incurred is -69% +_ 19% for one future calendar year followed by 0 for the remainder of a 30 year (future) period. Neither of these future forecast scenarios are remotely plausible; they result in answers around half of an optimistic scenario produces.

The Mack method does not capture calendar year trends, hardly has any predictive power, does not have any descriptors of the volatility in the data and it is unknown what calendar year trend assumption is made in forecasting - the Mack method does capture an average calendar year trend but there is no descriptor of it.

Indeed, we use the bootstrap technique to show that bootstrap samples from the Mack method are not related to the data and therefore the method has absolutely nothing to do with the features in the data.

As a result of the more recent calendar trend changes, a number of scenarios for the future are considered. As the next years' data become available, the most appropriate scenario can be selected. Until the data are available, a conservative approach is adapted. Naturally in practice the forecast scenario would be revisited at year end 2007 rather than waiting two years as for this study.

Dataset I

As for dataset H, both the (incremental) paid losses and the case reserve estimates (CREs) possess recent major shifts in calendar year trend suggesting that this may be driven by shifts in closure rates. However, the evidence of this hypothesis is not as strong here. The relationship between the two data types is suggestive of a hypothesised forecasting scenario going forward. This hypothesised scenario could be more fully tested if we had access to the number of closed claims (NCC) triangle.

The underlying calendar trend in the paid losses, as identified in the optimal PTF model, is 16.95% +_ 3.73% interrupted in 03-04 with a 65% +_ 11% trend change. This trend change is observed in the residuals of the Mack method, however the method is neither able to account for this change or quantify it.

For this dataset the Mack method applied to both the cumulative paid losses and the incurred losses gives mean answers that appear too high. However, if the trend in the paid losses reverts to 65% +_ 11% (the trend between 03-04), and then continues with most recent trend of 16.95% +_ 3.73% to calendar year 2036 then the Mack applied to the incurred data gives (only) a reasonable mean. However we argue in the body of the document that this scenario is pessimistic and quite unlikely.

The most likely scenario is to continue with the 16.95% +_ 3.73% trend. We would have more evidence to support this conclusion if we had the number of closed claims (NCC) triangle.

Composite model with capital allocation by line and calendar year

We can combine the four PTF models described above in one joint model in the MPTF framework. This modelling framework detects the process correlations between individual lines and fine-tunes that model parameters using them. The resulting Reserve Correlations are generally smaller than the corresponding process correlations but do have a significant effect on aggregate standard deviations and risk capital allocations. We give the highlights of such an analysis, under the assumption that **the four datasets correspond to four lines of business in the same company.**

Conclusion

In respect of a model belonging to the Probabilistic Trend Family (PTF) modelling framework the parameters estimates and process variability are depicted by four graphs. So if residuals do not have any structure, it is immediately clear that the trend structure in the data and the quality of the process variability have been fitted 'accurately' by the model. Moreover, the actuary has control on parameters (including calendar year trends) in formulating a forecasting scenario for the future.

By contrast, the Mack method does not have descriptors of the trend structure in the data (and does not model development period zero). It often lacks predictive power, and does not capture (and measure) calendar year trend changes. Moreover, often the weighted standardised residuals of Mack are skewed to the right as a result of large percentage variation on a log scale of the corresponding incremental data.

Our emphasis is not just on "ensuring" consistent estimates of prior year ultimates on updating, but also on the probability distributions of the paid losses by calendar year and their correlations for the purpose of computing the cost of capital.

Once each data set is updated, it is only in a probabilistic framework that forecast distributions as of 2006, can be compared with observed paid losses for 2007 and 2008. Updating, forecast tracking, and monitoring of the identified model is conducted in a probabilistic framework.

Data set D

Choice of Exposure Vector

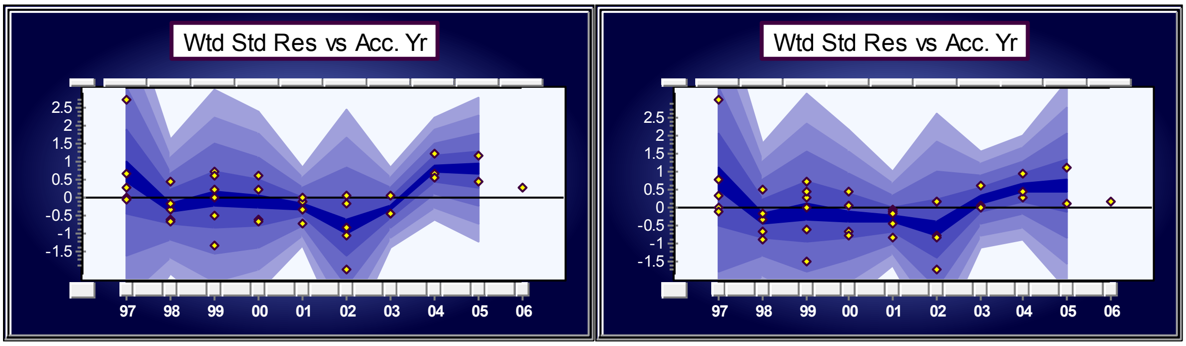
Accident year levels are affected by exposure such as number of policies, number of motor cars, wage roll, earned premium and so on. We normalize a loss development array by an accident year exposure vector. If there isn't one then the 'role' of exposures is taken up by the accident year level parameters in the identified model.

We can easily make a comparison between two competing exposure vectors by fitting the diagnostic Separation Method (SM) model to each normalized array. This SM model removes the average development period trends between any two contiguous development periods and any two contiguous accident years. The exposure vector that removes more of the resultant change in accident year levels is better.

Note that residuals always represent trends in data minus trends estimated by model. That is, residuals always represent the difference of two trends.

We illustrate this with dataset D. We have chosen 1997 as the first year, as no specific year is named in the original data. An exposure vector provided with the dataset, Earned Premium (EP), is tested against the alternative of using a uniform exposure vector (equivalently 'no exposure' since a uniform exposure has no changes in level).

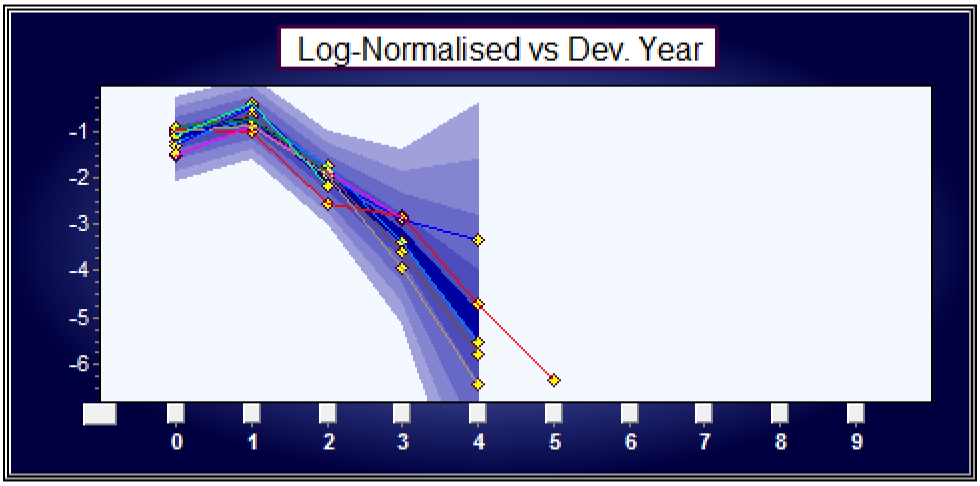
For ease of comparison, only the residuals versus accident year plots are shown in Figure 1; subsequent residual plots will also show the residuals versus development and calendar also.



The plot (above) on the right results from the dataset with premium as exposure. It appears to be slightly flatter than the residual display made without using an exposure measure particularly in the more recent accident years. Therefore, we will use the premium as exposure for this analysis.

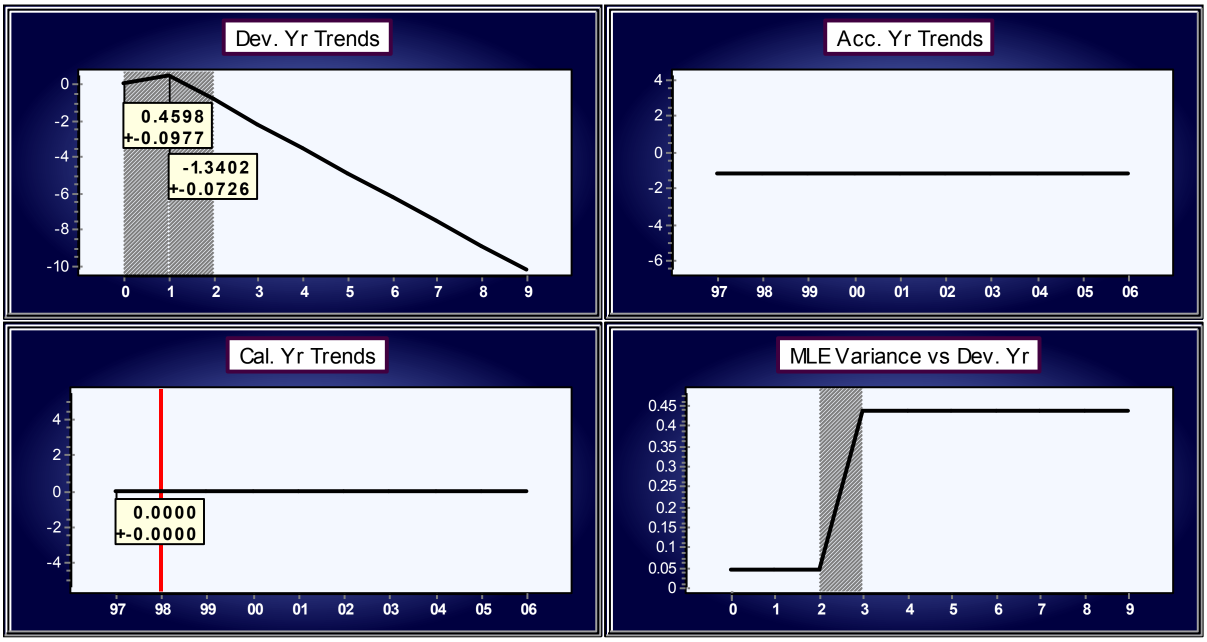
The identified PTF model for the paid losses

The data display below shows the losses by development year on a log scale after normalising by exposure.



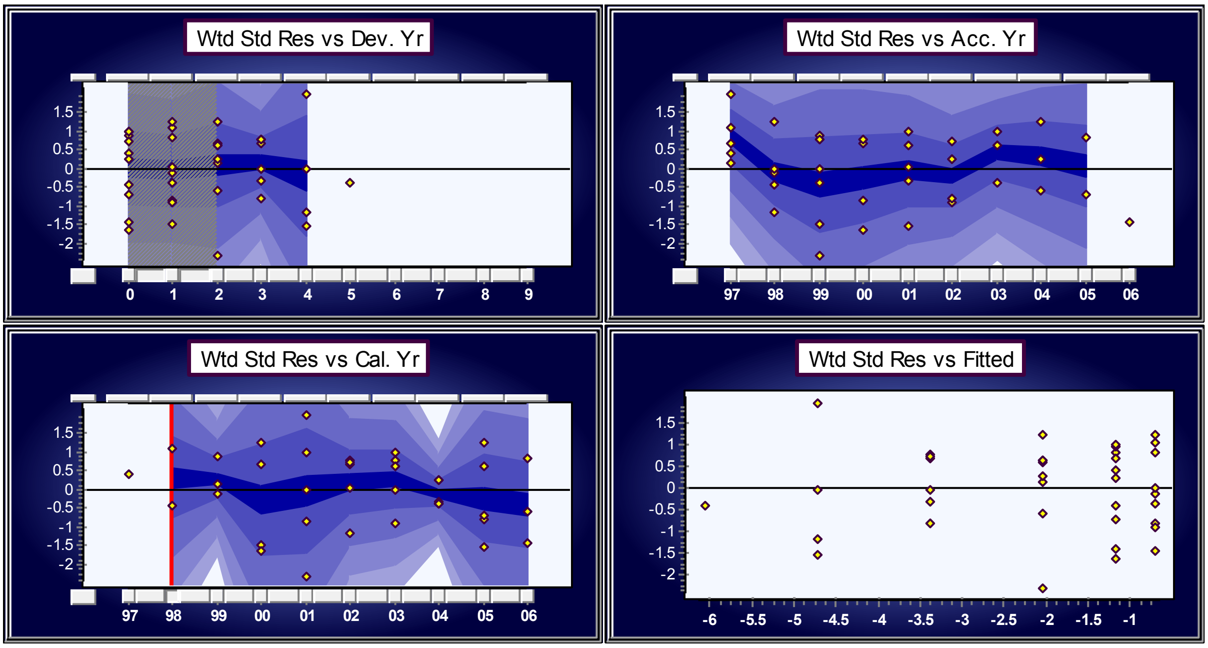
On the basis of the ten years of data available it is possible to infer that there is an increase from development period 0 to 1, followed by a strong decline in losses beginning after the second development year.

The model display, shown below, illustrates the expected results. There is a strong positive trend of 0.46 +- 0.098 followed by a sharp decay of 1.34 +- 0.073. Although the process variance is low initially, the standard error of the parameters is quite high. This result is not unreasonable given the number of data points.



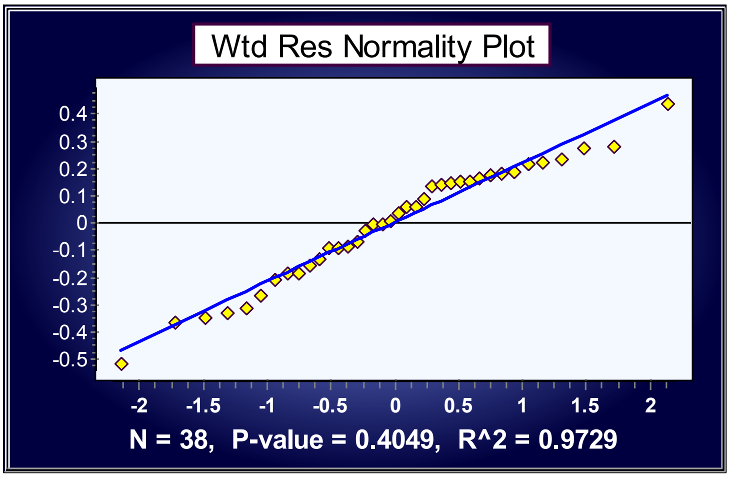
The model display above illustrates the trends found in three directions (development period, accident period, and calendar period), along with the process variance (of normal distributions) around the trend structure, versus development (lower right plot). Normality of the weighted standardized residuals is shown on the following page - it is an integral component of the model.

The model trend structure is very simple for this dataset, the only trend changes are by development direction and the accident and calendar year directions both show no changes. The process variance (around the trend structure) increases in development 2-3. This is not unexpected since on a percentage scale the observed values are generally more variable (volatile) the lower the mean is.



The residuals above show the remaining variation in the data after the model trends have been fitted. The residuals above are randomly scattered – there is no detectable structure remaining in the data.

Some model diagnostics consider the residuals (that equal trends in the data minus the trends estimated by the model), in the three directions and in relation to fitted values. Note the drop in the early accident period and the apparent increase from 2003 onward. Level changes at these points are statistically insignificant and are not included in the final model.



The P-Value for normality is high.

Forecasting with the identified PTF model and validation analyses

The identified (optimal) PTF model fits a normal distribution on a log scale to each cell and a log normal on a dollar scale. The trend structure relates means of the normal distributions.

Forecast distributions (normal on log scale and lognormal on dollar scale) for each cell are based on explicit assumptions, namely, development and calendar period trend estimates, accident year level parameter estimates (differences are trends), uncertainties thereof and the process variances (of the normal distributions).

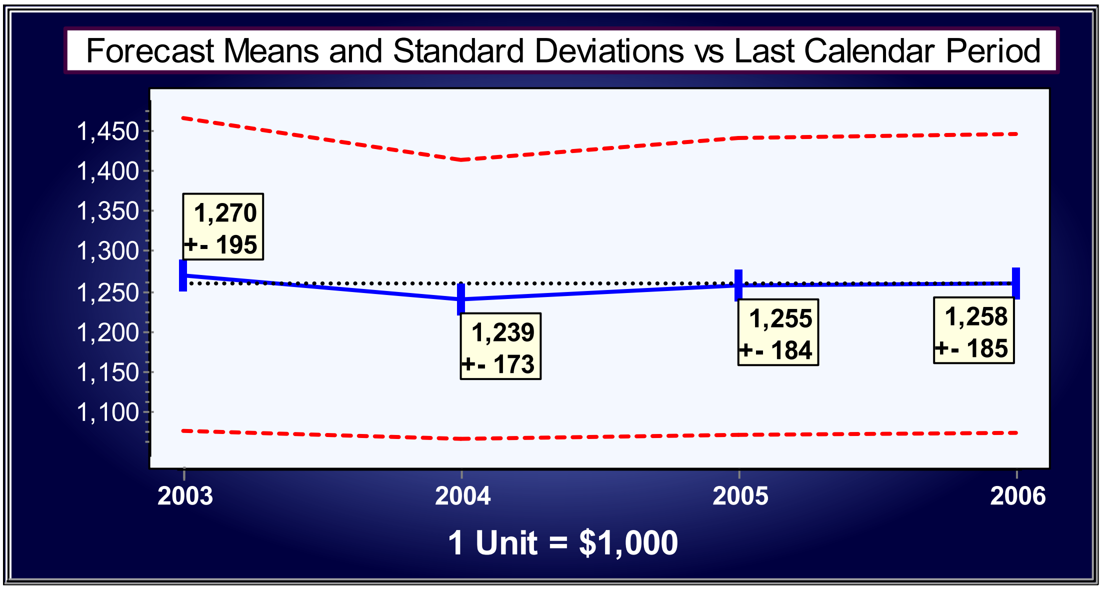
Forecast development periods can extend beyond the last development period in the triangle and accident periods- the latter for pricing and assessment of underwriting risk.

Usually, one would forecast with the most recently observed trend estimates and their uncertainties, but if additional information is known about the next year(s), then the forecast scenario may be adjusted to include this additional information.

The calendar trend of zero has been stable over the entire 10 year period, and therefore the future calendar trend was left unmodified. See validation analyses below.

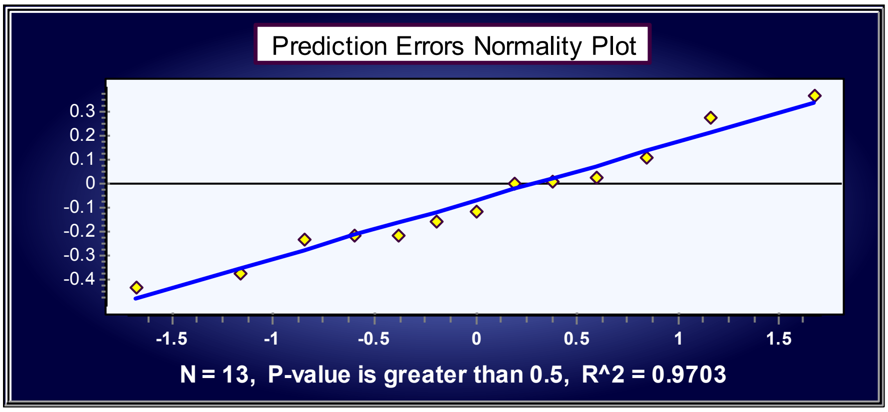
If there is information from other data types (see analyses of CRE below) and/or (external) information that this may not be the case (see datasets H & I), then this critical assumption can be modified.

Validation analyses involve removal of calendar years. We project the mean and standard deviation of total reserves (completion of square) beyond 2006 at end year 2006, 2005, 2004 and 2003.



In the validation graph above, the projected total reserve mean and standard deviation is compared for the model estimated at year end 2003, 2004, 2005 and 2006. For example, the mean and standard deviation of the total reserve beyond 2006, based on the model estimated at year end 2003 are 1,270,000, and 195,000 respectively.

If we were at year end 2003 we would have projected essentially the same reserve distribution compared to the calculation using all the data up to year end 2006 (conditional on knowing the exposures for the excluded years). This is indicative of calendar year trend of zero, constant accident year level, and the decay from development period 1-9 being stable.



Moreover, the normality plot above indicates that at year end 2003 the estimated model would have accurately predicted the volatility of the paid losses in the next four years.

The validation shows that the accident year (constant level), the development period decay, and the zero calendar trend are relatively stable.

In this example the forecast is extended by one development period. Due to the strong development parameter decay this extension has minimal effect on the ultimates.

Accident Period vs Development Period

	Cal. Per. Total	0	1	2	3	4	5	6	7	8	9	10	Reserve	Ultimate
1997	148	148	235	61	20	5	1	0	0	0	0	0	0	551
	158	158	290	62	25	16	0	0	0	0	0	0	0	0
1998	398	163	259	68	22	6	2	0	0	0	0	0	0	496
	435	145	245	87	17	2	0	0	0	0	0	0	0	0
1999	503	182	289	76	24	6	2	0	0	0	0	0	0	501
	522	215	203	44	33	5	1	0	0	0	0	0	0	0
2000	580	203	322	84	27	7	2	1	0	0	0	0	0	530
	453	138	261	95	36	0	0	0	0	0	0	0	0	0
2001	636	212	335	88	28	7	2	1	0	0	0	0	1	705
	595	257	329	98	18	2	0	0	0	0	0	0	0	0
2002	651	200	317	83	27	7	2	0	0	0	0	0	3	582
	687	228	252	86	12	1	2	0	0	0	0	0	2	2
2003	628	187	297	78	25	7	2	0	0	0	0	0	9	589
	619	228	266	87	-1	5	1	0	0	0	0	0	5	5
2004	764	347	549	144	46	12	3	1	0	0	0	0	63	1,252
	729	358	708	123	35	9	3	1	0	0	0	0	37	37
2005	1,052	389	616	161	52	14	4	1	0	0	0	0	231	1,277
	1,133	324	722	36	39	11	3	1	0	0	0	0	56	56
2006	1,231	437	692	181	58	15	4	1	0	0	0	0	952	1,263
	1,156	311	157	41	44	12	3	1	0	0	0	0	169	169
	Total Fitted/Paid		2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	Total Reserve	Total Ultimate
Cal. Per.	6,591		908	247	75	20	5	1	0	0	0	0	1,258	7,745
Total	6,487		165	59	45	13	4	1	0	0	0	0	185	185

1 Unit = $1,000

In the forecast table above a black value is the mean of the fitted (or projected) lognormal distribution for that cell, a blue value an observation, and the red value represents the standard deviation of the projected lognormal for the cell. The burgundy numbers in the row and column margins are standard deviations of sums of lognormals by accident year and calendar year.

Accident Yr Summary

Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Reserve	Ultimate	Dev.	Reserve	Ultimate	Std.Dev.|Data	+-Ult|Data
1997	0	551	0	1.12	0.00	0	0
1998	0	496	0	0.92	0.00	0	0
1999	0	501	0	0.82	0.00	0	0
2000	0	530	0	0.75	0.00	0	0
2001	1	705	0	0.69	0.00	0	0
2002	3	582	2	0.65	0.00	0	2
2003	9	589	5	0.61	0.01	1	5
2004	63	1,252	37	0.58	0.03	10	35
2005	231	1,277	56	0.24	0.04	40	39
2006	952	1,263	169	0.18	0.13	61	158
Total	1,258	7,745	185	0.15	0.02	75	169

1 Unit = $1,000

The above summary table gives reserve and ultimate means, standard deviations and CVs by accident year and total.

How do we know if prior year estimates of ultimates are consistent on updating?

The two columns on the right provide some statistics conditional on the next calendar year's (2007) data (not observed yet).

If on updating the model using 2007 data there is no significant change in calendar year (zero) trend and no significant change in decay trend from period 1-9, then the mean ultimate of all the mean conditional on 2007 data ultimates, is the mean ultimate as at end 2006.

The second column from the right represents on average the Standard Deviation (SD) of ultimate given 2007 data (note reduction due decrease in parameter uncertainty with more data and forecasting horizon not as far), whereas the column on extreme right represents the SD of the conditional on 2007 expectation of mean ultimate. It gives an idea of possible statistical variation in mean ultimate that maintains consistent estimates of prior ultimates on update.

There is expected to be little change in the re estimation of prior year ultimates assuming that the trend applied in the forecast holds true for the next calendar year(s).

If, for some reason, the trend was not zero, then on updating and monitoring of the model based on next calendar year's data, adjustments could be made and forecast scenario possibly amended.

As a test for reasonableness, the projected future liability stream can be compared to the paid losses in the more recent calendar years. This comparison can be made directly from the previous forecast table, but a second table is produced below. This comparison table, by excluding development periods in the last calendar year, allows a direct comparison between the most recently observed paid losses and the projected future paid losses.

The table shows that the projected mean paid losses are "in line" with the amounts paid in the previous year, given the increases in exposure more recently by accident year.

Comparison of Last Cal. Yr with First Three Future Cal. Yrs

Excl. Dev. Year	Last Calendar Year (2006)	Calendar Year	Forecast Mean	Forecast Std dev
0:0	845	2007	908	165
0:1	123	2008	247	59
0:2	0	2009	75	45

1 Unit = $1,000

2004	764	347	549	144	46
	729	358	708	123	35
2005	1,052	389	616	161	52
	1,133	324	722	36	39
2006	1,231	437	692	181	58
	1,156	311	157	41	44
	Total Fitted/Paid		2007	2008	2009
Cal. Per.	6,591		908	247	75
Total	6,487		165	59	45

1 Unit = $1,000

The liability stream is also reasonable from the forecast table excerpt shown above. The projected mean of 908k compares favourably with the last payment of 1,156k given that the 908k does not include any payments in development zero and exposures by accident year increase.

Incurred Losses

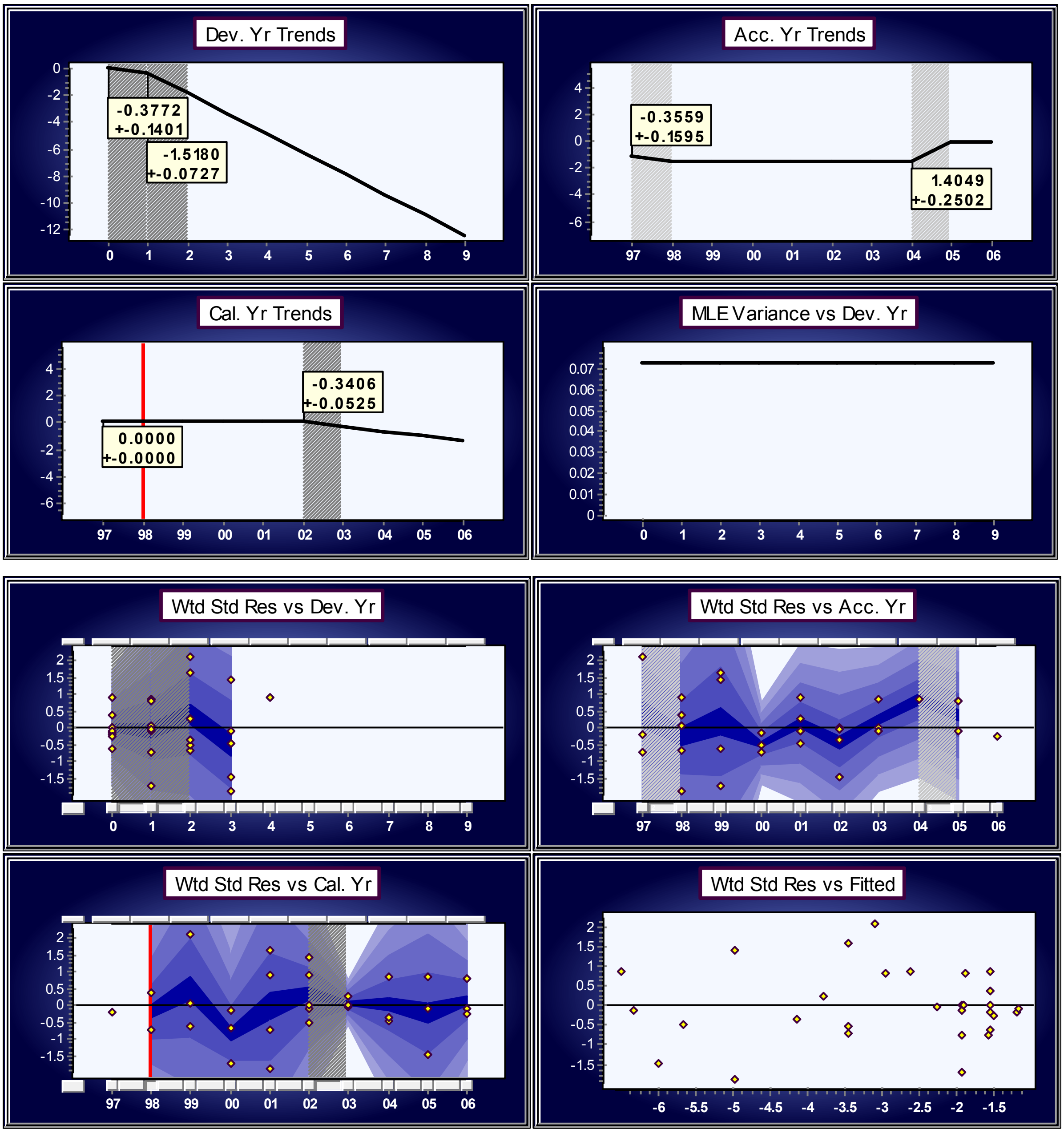
Acc. Yr	Premium	Paid To 2006	Incurred To 2006	CRE 2006	Mean Reserve	Mean Ultimate	Standard Dev.
1997	460	551	551	0	0	551	0
1998	507	496	496	0	0	496	0
1999	566	501	501	0	0	501	0
2000	631	530	530	0	0	530	0
2001	657	704	704	0	1	705	0
2002	622	579	579	0	3	582	2
2003	581	580	581	1	9	589	5
2004	1,076	1,189	1,192	3	63	1,252	37
2005	1,207	1,046	1,278	232	231	1,277	56
2006	1,356	311	586	275	952	1,263	169
Total	7,663	6,487	6,998	511	1,258	7,745	185

1 Unit = $1,000

The summary table above includes both the premium, incurred to date, and CRE.

Case Reserve Estimates (CREs)

The case reserve estimates were modelled for comparison with the Paid Losses. Do CREs lag or lead paid losses in respect of trends?



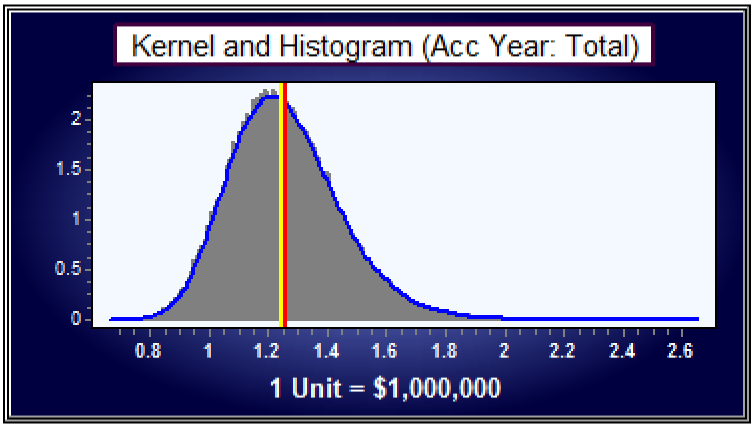
The increase between 2004 and 2005 is slightly higher than the negative calendar trend over the last four years. The case reserve estimates in 2005 are unusually high in development period 1 – this would need to be investigated.

There does not appear to be any evidence from the trend structure in the CREs of an impending change in calendar year trend of zero in the paid losses.

Reserve probability distributions by calendar year, accident year and total

The identified model in conjunction with the forecast scenario projects log normal distributions for each cell and their correlations.

Since there is no closed analytical form for the sum of log-normal distributions, we simulate from the correlated lognormals to find distributions of aggregates.



This histogram is based on 100,000 draws from the joint distribution of the forecast cells. The red line indicates the mean and the yellow the median.

The total reserve distribution is only moderately skewed.

Quantile Statistics, VaR and T-VaR (Acc Year: Total)

	Sample				Kernel			
	Quantile	# S.D.'s	VaR	T-VaR	Quantile	# S.D.'s	VaR	T-VaR
99.5	1.824	3.054	0.566	0.661	1.835	3.112	0.577	0.667
99.4	1.811	2.982	0.553	0.644	1.817	3.013	0.558	0.654
99.3	1.793	2.888	0.535	0.630	1.801	2.927	0.543	0.638
99.2	1.776	2.795	0.518	0.617	1.787	2.853	0.529	0.627
99.1	1.765	2.734	0.507	0.605	1.775	2.786	0.516	0.614
99.0	1.758	2.699	0.500	0.595	1.764	2.727	0.505	0.602
98.0	1.684	2.296	0.426	0.526	1.687	2.313	0.429	0.529
97.0	1.630	2.005	0.372	0.483	1.640	2.062	0.382	0.489
96.0	1.601	1.852	0.343	0.451	1.609	1.893	0.351	0.460
95.0	1.578	1.725	0.320	0.427	1.585	1.764	0.327	0.432
94.0	1.560	1.628	0.302	0.408	1.565	1.658	0.307	0.414
93.0	1.546	1.554	0.288	0.392	1.548	1.564	0.290	0.393
92.0	1.530	1.465	0.272	0.378	1.532	1.480	0.274	0.380
91.0	1.515	1.384	0.257	0.365	1.518	1.402	0.260	0.368
90.0	1.503	1.321	0.245	0.354	1.505	1.329	0.246	0.355
89.0	1.489	1.243	0.230	0.343	1.492	1.261	0.234	0.346
88.0	1.477	1.181	0.219	0.333	1.480	1.197	0.222	0.335

Mean = 1.258, S.D. = 0.185, Provision = 1.258, 1 Unit = $1,000,000

The Sample columns in the percentile table excerpt above refer to the sample of 100,000 draws from the joint distribution of forecast cells, the Kernel columns refer to a smoothed version of the empirical distribution. The VaR and T-VaR figures are based on reserving at the mean; they can be recomputed by a shift if a different figure is used for the reserve.

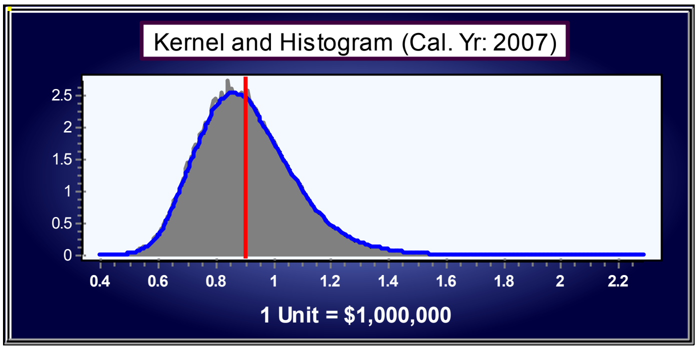
The simulations from the correlated lognormals allow for aggregate distributions of the total reserve (as shown above), but also can be used for aggregates of calendar years or accident years.

By way of example, the distribution of the next calendar year and the sum of the next two calendar years are computed.

Quantile Statistics, VaR and T-VaR (Cal. Yr: 2007)

%	Sample				Kernel			
	Quantile	# S.D.'s	VaR	T-VaR	Quantile	# S.D.'s	VaR	T-VaR
99.5	1.936	3.061	0.590	0.685	1.938	3.074	0.593	0.688
99.4	1.916	2.956	0.570	0.667	1.918	2.971	0.573	0.670
99.3	1.897	2.860	0.552	0.652	1.902	2.885	0.556	0.657
99.2	1.882	2.782	0.537	0.639	1.888	2.812	0.542	0.644
99.1	1.871	2.727	0.526	0.627	1.876	2.749	0.530	0.632
99.0	1.862	2.676	0.516	0.616	1.865	2.693	0.519	0.619
98.0	1.789	2.302	0.444	0.546	1.792	2.315	0.447	0.548
97.0	1.743	2.064	0.398	0.504	1.746	2.079	0.401	0.507
96.0	1.710	1.890	0.365	0.473	1.713	1.906	0.368	0.476
95.0	1.684	1.758	0.339	0.449	1.686	1.768	0.341	0.451
94.0	1.661	1.638	0.316	0.429	1.664	1.653	0.319	0.431
93.0	1.643	1.544	0.298	0.411	1.645	1.554	0.300	0.413
92.0	1.626	1.455	0.281	0.396	1.628	1.467	0.283	0.398
91.0	1.611	1.378	0.266	0.382	1.613	1.389	0.268	0.384
90.0	1.598	1.309	0.252	0.370	1.600	1.319	0.254	0.372
89.0	1.586	1.246	0.240	0.359	1.587	1.254	0.242	0.360
88.0	1.574	1.186	0.229	0.348	1.576	1.194	0.230	0.350

Mean = 1.345, S.D. = 0.193, Provision = 1.345, 1 Unit = $1,000,000



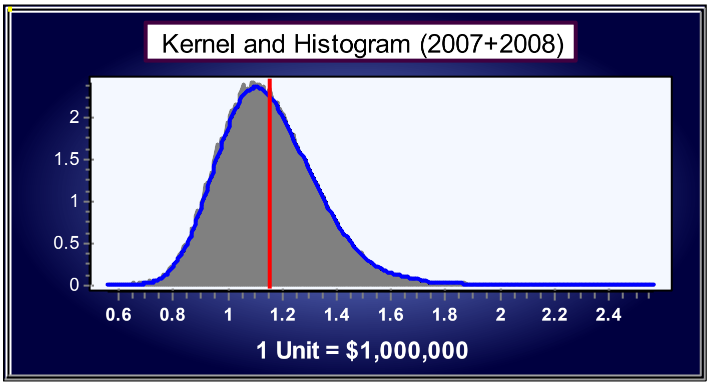
Based on the forecast assumptions specified, the loss distribution for 2007 is simulated as displayed above. The minimum loss simulated was around 400k whereas the maximum was around 2.2M; a wide range. On average, however, we would expect around 908K; the aggregate mean of the projected cells for the 2007 calendar year.

The simulation was run again – this time for the aggregate of the next two calendar years.

Quantile Statistics, VaR and T-VaR (2007+2008)

%	Sample				Kernel			
	Quantile	# S.D.'s	VaR	T-VaR	Quantile	# S.D.'s	VaR	T-VaR
99.5	3.563	2.930	0.841	0.967	3.569	2.950	0.847	0.973
99.4	3.539	2.846	0.817	0.944	3.544	2.863	0.822	0.948
99.3	3.517	2.769	0.795	0.924	3.523	2.789	0.801	0.930
99.2	3.499	2.707	0.777	0.907	3.504	2.723	0.782	0.912
99.1	3.482	2.648	0.761	0.892	3.487	2.664	0.765	0.896
99.0	3.466	2.591	0.744	0.878	3.472	2.612	0.750	0.883
98.0	3.365	2.241	0.644	0.783	3.369	2.255	0.648	0.787
97.0	3.303	2.023	0.581	0.726	3.307	2.037	0.585	0.730
96.0	3.258	1.868	0.536	0.684	3.261	1.878	0.539	0.687
95.0	3.222	1.742	0.500	0.650	3.225	1.751	0.503	0.653
94.0	3.190	1.631	0.468	0.623	3.194	1.644	0.472	0.626
93.0	3.164	1.538	0.442	0.599	3.167	1.550	0.445	0.601
92.0	3.140	1.458	0.419	0.578	3.143	1.468	0.422	0.580
91.0	3.120	1.387	0.398	0.559	3.122	1.394	0.400	0.561
90.0	3.100	1.317	0.378	0.542	3.102	1.325	0.381	0.543
89.0	3.081	1.252	0.360	0.526	3.084	1.261	0.362	0.528
88.0	3.065	1.195	0.343	0.512	3.067	1.202	0.345	0.513

Mean = 2.722, Sample S.D. = 0.287, Provision = 2.722, 1 Unit = $1,000,000



Again, the distribution has a wide range of possible outcomes based on the model and forecast. The minimum loss simulated for the sum of the next two periods being roughly 600K and the largest simulated loss around 2.5M. On average, we expect a figure around 1.2M – the sum of the means of the payments for these two calendar years.

Summary by calendar years required for cost of capital calculations

The means and standard deviations of the probability distributions of paid losses by calendar year can be extracted from the forecast table.

Calendar Yr Summary				
Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve	Cum. Payment as % of total
2007	908	165	0.18	72.19
2008	247	59	0.24	91.84
2009	75	45	0.60	97.84
2010	20	13	0.63	99.42
2011	5	4	0.66	99.85
2012	1	1	0.71	99.96
2013	0	0	0.77	99.99
2014	0	0	0.84	100.00
2015	0	0	0.93	100.00
2016	0	0	1.12	100.00
Total	1,258	185	0.15	100.00
1 Unit = $1,000				

These projections assume that no further business is written and purely projects the liability stream (for the reserving risk). The expected payments in the next two years, in the triangle without projecting further underwriting years, are 908,000 +- 165,000 and 247,000 +- 59,000 respectively. Note that the distribution is not symmetric and the +- component refers to the standard error of the distribution.

The identified PTF can also be used to project future underwriting years as well as reserving years. In order to facilitate this, it is assumed that there are no material premium changes for 2007 and 2008 (the relative exposure is comparable to 2006). Therefore, the level and exposure changes can be maintained for these two periods.

Accident Yr Summary							
Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev.|Data	+-Ult|Data
2007	1,389	1,389	196	0.14	0.14	168	101
2008	1,389	1,389	196	0.14	0.14	195	21
Total	2,778	2,778	290	0.10	0.10	266	115
1 Unit = $1,000							

Calendar Yr Summary				
Calendar Yr	**Mean Outstanding**	**Standard Dev.**	**CV Outstanding**	**Cum. Payment as % of total**
2007	437	99	0.23	15.73
2008	1,129	185	0.16	56.38
2009	873	163	0.19	87.81
2010	239	61	0.25	96.41
2011	73	46	0.62	99.05
2012	19	13	0.65	99.75
2013	5	4	0.68	99.93
2014	1	1	0.73	99.98
2015	0	0	0.78	100.00
2016	0	0	0.85	100.00
2017	0	0	0.92	100.00
2018	0	0	1.12	100.00
Total	2,778	290	0.10	100.00

1 Unit = $1,000

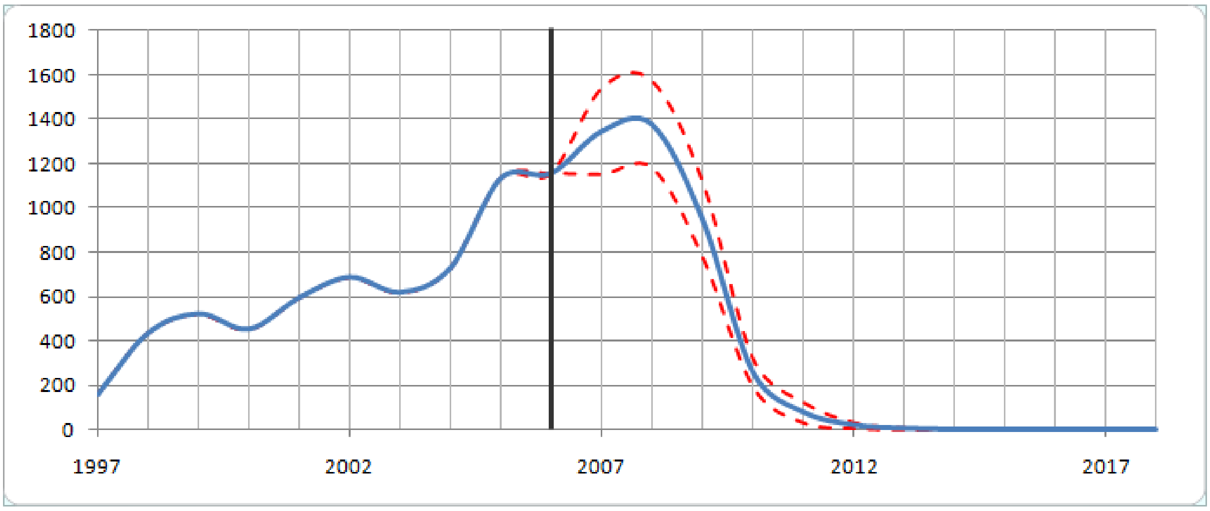
A combined reserve and underwriting table is then computed and is displayed below.

Accident Period vs Development Period														
	Cal. Per. Total	0	1	2	3	4	5	6	7	8	9	10	Outstanding	Ultimate
1997	148	148	235	61	20	5	1	0	0	0	0	0	0	551
	158	158	290	62	23	16	0	0	0	0	0	0	0	0
1998	398	143	259	68	22	6	2	0	0	0	0	0	0	496
	435	145	245	87	17	2	0	0	0	0	0	0	0	0
1999	503	182	289	76	24	6	2	0	0	0	0	0	0	501
	522	215	203	44	33	5	1	0	0	0	0	0	0	0
2000	580	203	323	84	27	7	2	1	0	0	0	0	0	530
	453	138	261	95	36	0	0	0	0	0	0	0	0	0
2001	636	212	335	88	28	7	2	1	0	0	0	0	1	705
	585	257	329	98	18	2	0	0	0	0	0	0	0	0
2002	651	300	317	83	27	7	2	0	0	0	0	0	3	582
	687	228	252	86	12	1	2	0	0	0	0	0	2	2
2003	628	187	297	78	25	7	2	0	0	0	0	0	3	583
	613	228	266	87	-1	5	1	0	0	0	0	0	5	5
2004	764	347	549	144	46	12	3	1	0	0	0	0	63	1,252
	729	338	708	123	33	9	3	1	0	0	0	0	37	37
2005	1,052	389	616	161	52	14	4	1	0	0	0	0	231	1,277
	1,133	324	722	39	39	11	3	1	0	0	0	0	56	56
2006	1,231	437	692	181	58	15	4	1	0	0	0	0	932	1,263
	1,156	311	157	41	44	12	3	1	0	0	0	0	169	169
2007	1,345	437	692	181	58	15	4	1	0	0	0	0	1,389	1,389
	193	99	157	41	44	12	3	1	0	0	0	0	196	196
2008	1,376	437	692	181	58	15	4	1	0	0	0	0	1,389	1,389
	195	99	157	41	44	12	3	1	0	0	0	0	196	196
			2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	Total Reserve	Total Ultimate
Cal. Per. Total			349	258	79	21	6	1	0	0	0	0	4,036	10,523
			169	63	46	13	4	1	0	0	0	0	362	362

1 Unit = $1,000

Calendar Yr Summary				
Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve	Cum. Payment as % of total
2007	1,345	193	0.14	33.33
2008	1,376	195	0.14	67.43
2009	949	169	0.18	90.94
2010	259	63	0.24	97.35
2011	79	46	0.59	99.30
2012	21	13	0.61	99.81
2013	6	4	0.65	99.95
2014	1	1	0.70	99.99
2015	0	0	0.75	100.00
2016	0	0	0.82	100.00
2017	0	0	0.92	100.00
2018	0	0	1.12	100.00
Total	4,036	362	0.09	100.00
1 Unit = $1,000				

The projected mean payments in 2007 and 2008 are now 1,345,000 and 1,376,000 and include the projection for the first development period (0). The projected means are higher than those observed to date due to the increasing exposure.



A comparison with the mean paid to date by calendar period is made above with the projected calendar year means incorporating the next two underwriting periods. The dashed red line represents the mean +- one standard deviation. The projections are consistent with the paid-to-date thus far and are assuming no changes in exposure other trend or level changes in the model.

Accident Yr Summary							
Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev.|Data	+-Ult|Data
1997	0	551	0	1.12	0.00	0	0
1998	0	496	0	0.92	0.00	0	0
1999	0	501	0	0.82	0.00	0	0
2000	0	530	0	0.75	0.00	0	0
2001	1	705	0	0.69	0.00	0	0
2002	3	582	2	0.65	0.00	0	2
2003	9	589	5	0.61	0.01	1	5
2004	63	1,252	37	0.58	0.03	10	35
2005	231	1,277	56	0.24	0.04	40	39
2006	952	1,263	169	0.18	0.13	61	158
2007	1,389	1,389	196	0.14	0.14	168	101
2008	1,389	1,389	196	0.14	0.14	195	21
Total	4,036	10,523	362	0.09	0.03	281	228

1 Unit = $1,000

The table above is consistent with the previously shown accident year summary, but includes the additional two rows for the expected figures of the underwriting risk in 2007 and 2008. In the PTF modelling framework, the projection of reserving and underwriting risk are not seen as two separate problems but rather the two risks can (and should) be treated as liability risk.

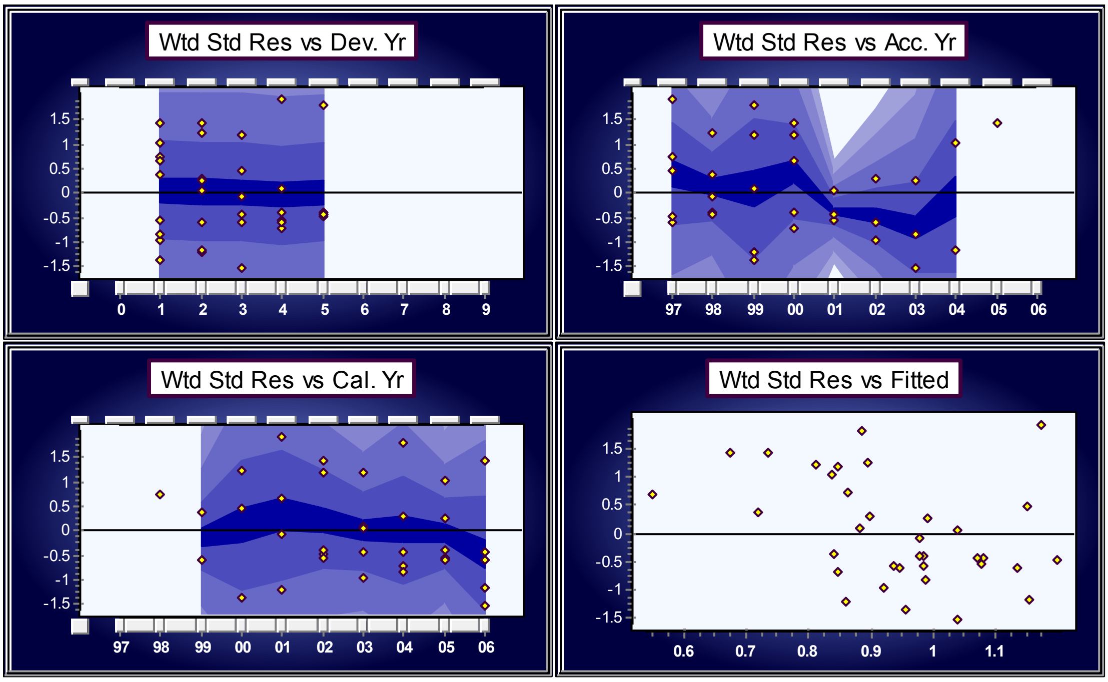
Note that on average, when the next calendar year is added, the standard deviations of the outstanding amounts reduce. Knowing the next calendar year reduces the amount of volatility in the ultimates since you've taken out the uncertainty associated with the next calendar year and the forecasting horizon is one year shorter.

The Mack Method

The Mack Method was applied to the Cumulative Paid Losses and the (Cumulative) Incurred Losses. The Mack method is a regression (through the origin) formulation of volume weighted average link ratios.

Mack Method applied to Paid Loss data

The Mack Method was fitted to the cumulative Paid Loss data and the residuals plotted against each direction and the fitted values. The plots are shown below.



The residuals above indicate that the method is over fitting the more recent accident years. The lower residuals all happen to be positive (lower right), however there are too few of them to assign any meaning to this.

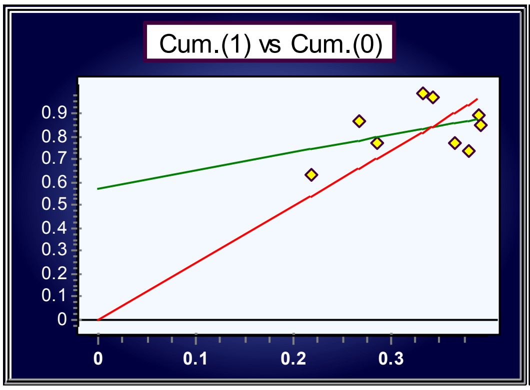
The Mack method makes two assumptions:

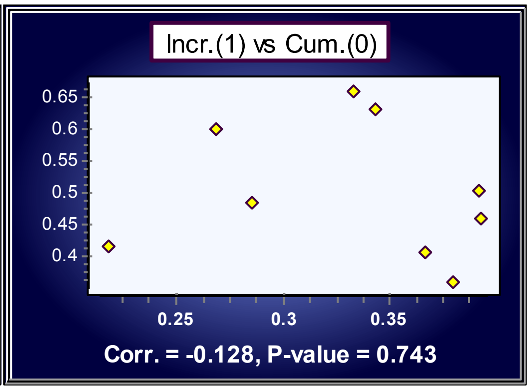
- Cumulatives in a development period versus cumulatives in the previous period go through the origin;
- Incrementals in a development period versus the cumulatives in the previous period are significantly correlated.

If the first assumption is not satisfied the Murphy (1994) method that includes an intercept is better.

If the second assumption is not satisfied then the Mack method does not have any predictive power. [See the paper by Barnett and Zehnwirth (2000). This paper is prescribed reading for Study 6 of the CAS].

Both the above assumptions are not satisfied for these data between any two consecutive development periods. This is demonstrated graphically.





The above plots show that you need an intercept and there is no relationship between the incrementals at period 1 versus the cumulatives at period 0 (right hand plot).

The above findings apply to each pair of consecutive development periods.

Incremental Forecasts

	0	1	2	3	4	5	6	7	8	9	Outstanding	Ultimate
1997	158	239	74	19	5	0	0	0	0	0	0	551
	158	290	62	25	16	0	0	0	0	0	0	0
1998	145	219	64	8	4	0	0	0	0	0	0	496
	145	245	87	17	2	0	0	0	0	0	0	0
1999	215	325	69	17	4	0	0	0	0	0	0	501
	215	203	44	33	5	1	0	0	0	0	0	0
2000	138	209	65	19	5	0	0	0	0	0	0	530
	138	261	95	36	0	0	0	0	0	0	0	0
2001	257	389	96	26	6	0	0	0	0	0	0	704
	257	329	98	18	2	0	0	0	0	0	0	0
2002	228	345	79	21	5	0	0	0	0	0	0	579
	228	252	86	12	1	1	0	0	0	0	1	1
2003	228	345	81	22	5	0	0	0	0	0	5	585
	228	266	87	-1	7	1	0	0	0	0	7	7
2004	358	541	175	45	11	0	0	0	0	0	56	1,245
	358	708	123	31	15	1	0	0	0	0	35	35
2005	324	490	172	46	11	0	0	0	0	0	229	1,275
	324	722	49	34	16	1	0	0	0	0	64	64
2006	311	470	128	34	8	0	0	0	0	0	642	953
	311	175	53	31	14	1	0	0	0	0	220	220
		2007	2008	2009	2010	2011	2012	2013	2014	2015	Total Outstanding	Total Ultimate
Cal. Yr Totals	****	692	185	46	9	0	0	0	0	0	932	7,419
	****	****	****	****	****	****	****	****	****	****	236	236

1 Unit = $1,000

Accident Yr Summary

Acc. Yr	Mean Outstanding	Mean Ultimate	Standard Dev.	CV Outstanding	CV Ultimate
1997	0	551	0	****	****
1998	0	496	0	****	****
1999	0	501	0	****	****
2000	0	530	0	****	****
2001	0	704	0	****	****
2002	0	579	1	2.71	0.00
2003	5	585	7	1.38	0.01
2004	56	1,245	35	0.62	0.03
2005	229	1,275	64	0.28	0.05
2006	642	953	220	0.34	0.23
Total	932	7,419	236	0.25	0.03

1 Unit = $1,000

Calendar Yr Summary

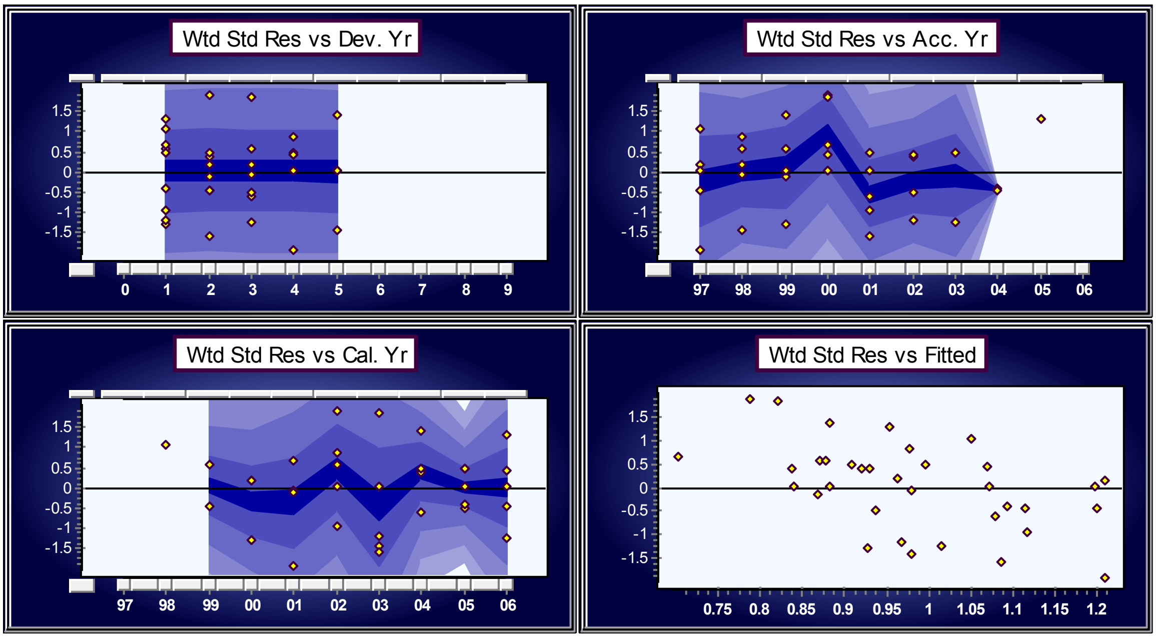
Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve
2007	692	****	****
2008	185	****	****
2009	46	****	****
2010	9	****	****
2011	0	****	****
2012	0	****	****
2013	0	****	****
2014	0	****	****
2015	0	****	****
To Ultimate	0	****	****
Total	932	236	0.25

1 Unit = $1,000

Mack Method applied to Incurred Loss data

We move onto examining the projections of the outstanding based on the Incurred Losses.

The Incurred Losses do not provide projections of the volatility in the paid losses by calendar year. The latter is required for computing the cost of capital. Nevertheless, for completeness, the calculations from the Mack Method (ratios were not a significant contributor of information), along with the residuals, are shown below.



Incremental Forecasts

	0	1	2	3	4	5	6	7	8	9	Outstanding	Ultimate
1997	289	195	28	7	-1	-0	0	0	0	0	0	551
	289	235	25	8	-6	0	0	0	0	0	0	0
1998	264	178	25	6	-1	-0	0	0	0	0	0	496
	264	200	26	6	1	-1	0	0	0	0	0	0
1999	314	212	25	6	-1	-0	0	0	0	0	0	501
	314	153	24	10	-1	1	0	0	0	0	0	0
2000	265	179	25	7	-1	-0	0	0	0	0	0	530
	265	207	39	19	0	0	0	0	0	0	0	0
2001	438	295	36	9	-2	-0	0	0	0	0	0	704
	438	240	22	4	0	0	0	0	0	0	0	0
2002	360	243	29	7	-1	-0	0	0	0	0	-0	579
	360	183	32	4	0	1	0	0	0	0	1	1
2003	315	212	29	7	-1	-0	0	0	0	0	-0	580
	315	234	33	-1	3	1	0	0	0	0	3	3
2004	703	474	61	15	-3	-0	0	0	0	0	16	1,205
	703	435	54	14	6	2	0	0	0	0	16	16
2005	688	463	68	17	-3	-0	0	0	0	0	314	1,360
	688	590	18	16	7	2	0	0	0	0	25	25
2006	586	395	52	13	-2	-0	0	0	0	0	733	1,044
	586	99	17	15	6	2	0	0	0	0	107	107
		2007	2008	2009	2010	2011	2012	2013	2014	2015	Total Outstanding	Total Ultimate
Cal. Yr Totals	****	477	67	10	-2	-0	0	0	0	0	1,063	7,550
	****	****	****	****	****	****	****	****	****	****	113	113

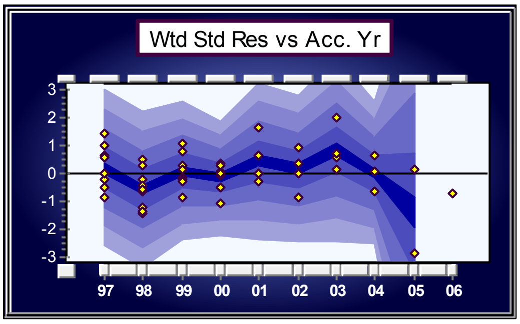
1 Unit = $1,000

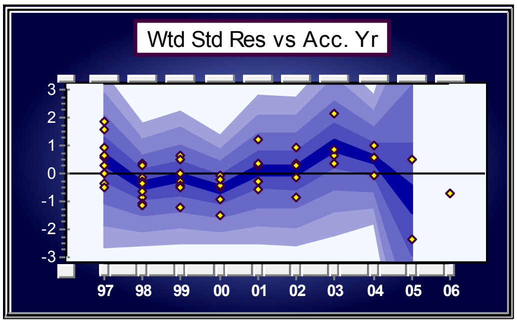
The above does not allocate the 511,000 CRE to the calendar liability stream. However, the projected total for 477,000 in 2007 could be expected to take a significant proportion of this unallocated amount.

Data set F

Choice of Exposure Vector

As with the previously analysed dataset, the first step in the modelling process is to analyse the exposure vectors: Earned Premium versus uniform (‘no exposure’).

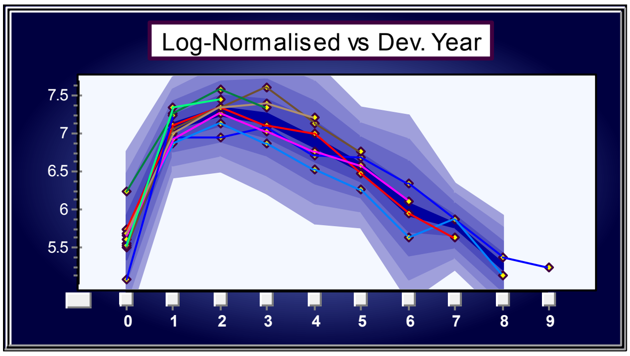




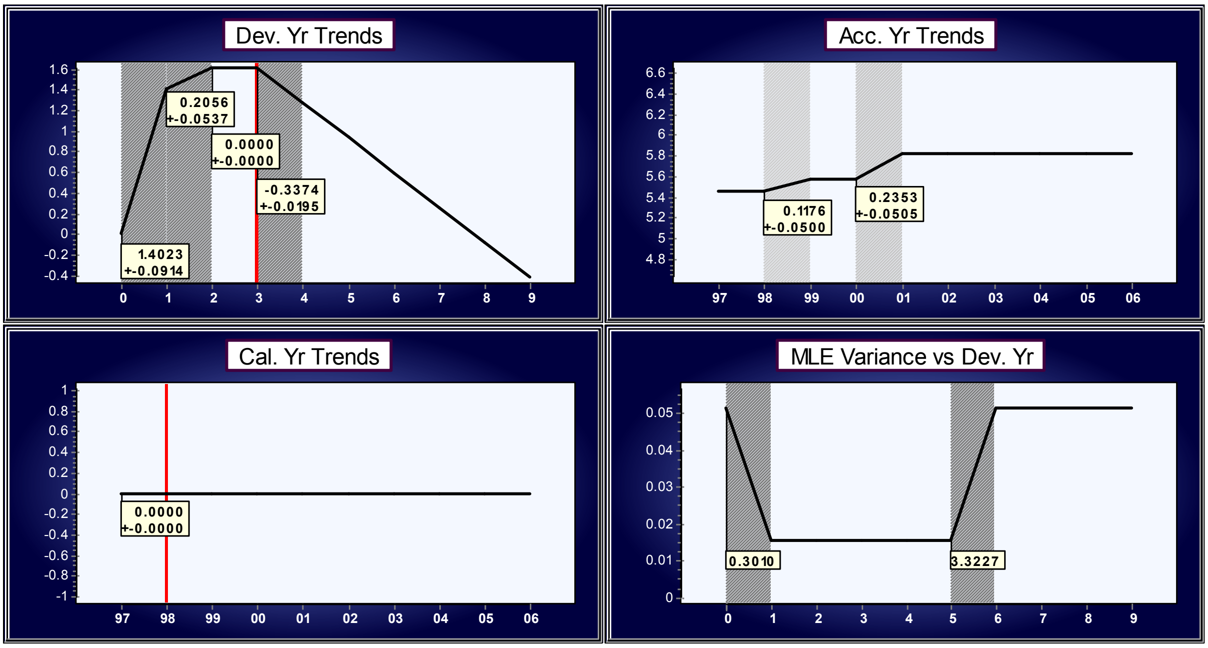
There is little difference between these two displays, however the dataset without exposure (left) is slightly ‘flatter’. It is better not to add redundant information into a model, so the dataset without exposure was used for the remainder of the analysis.

The identified PTF model for the paid losses

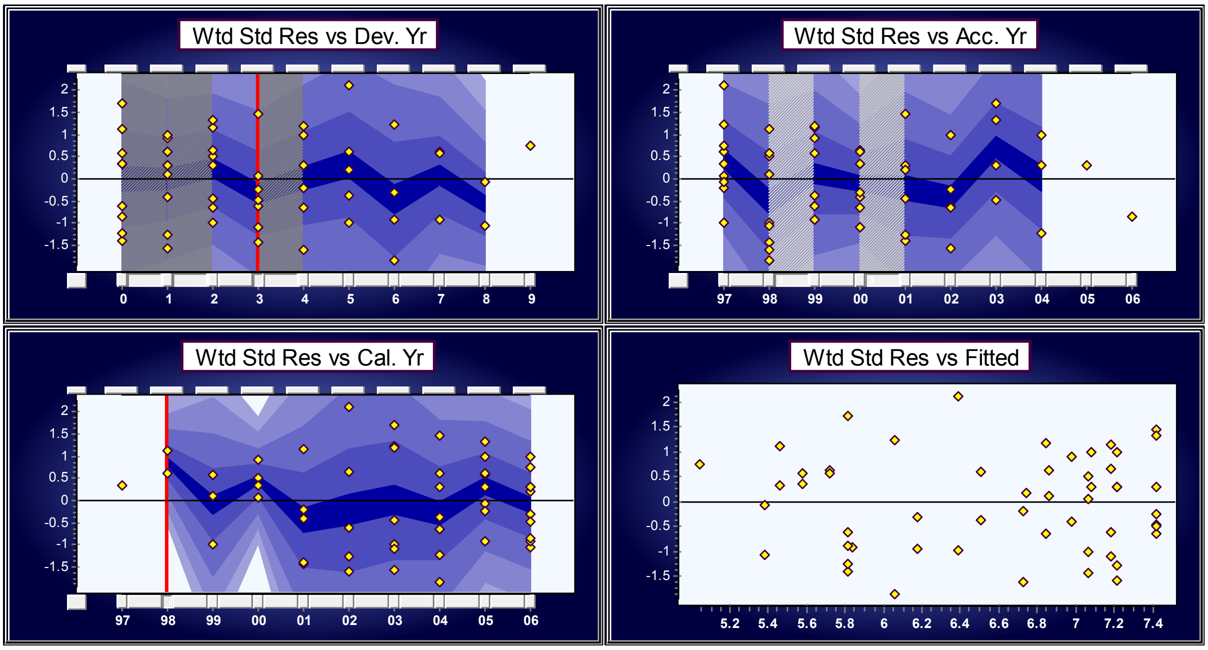
The data display below shows the losses by development year on a log scale after normalising by exposure.

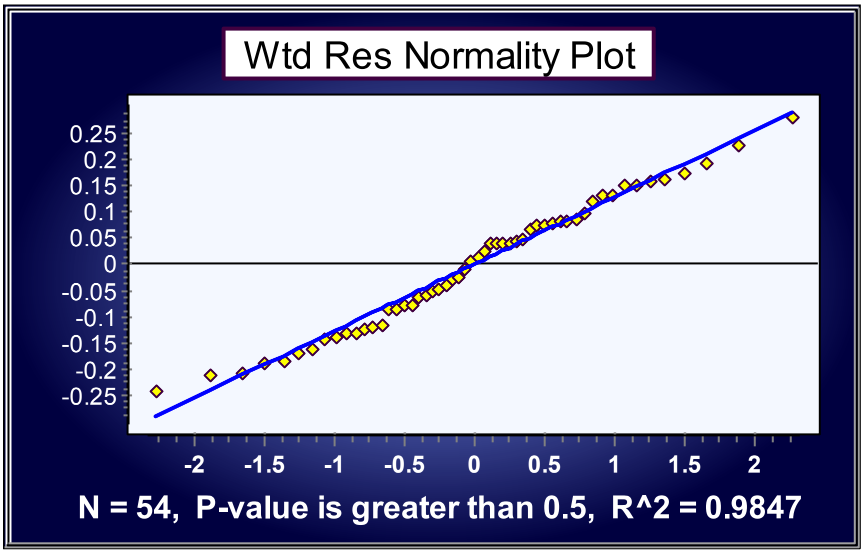


On the basis of the ten years of data available it is possible to infer that there is a strong increase from development period 0 to 1, followed by two more or less flat periods before losses begin to decline in development 3.



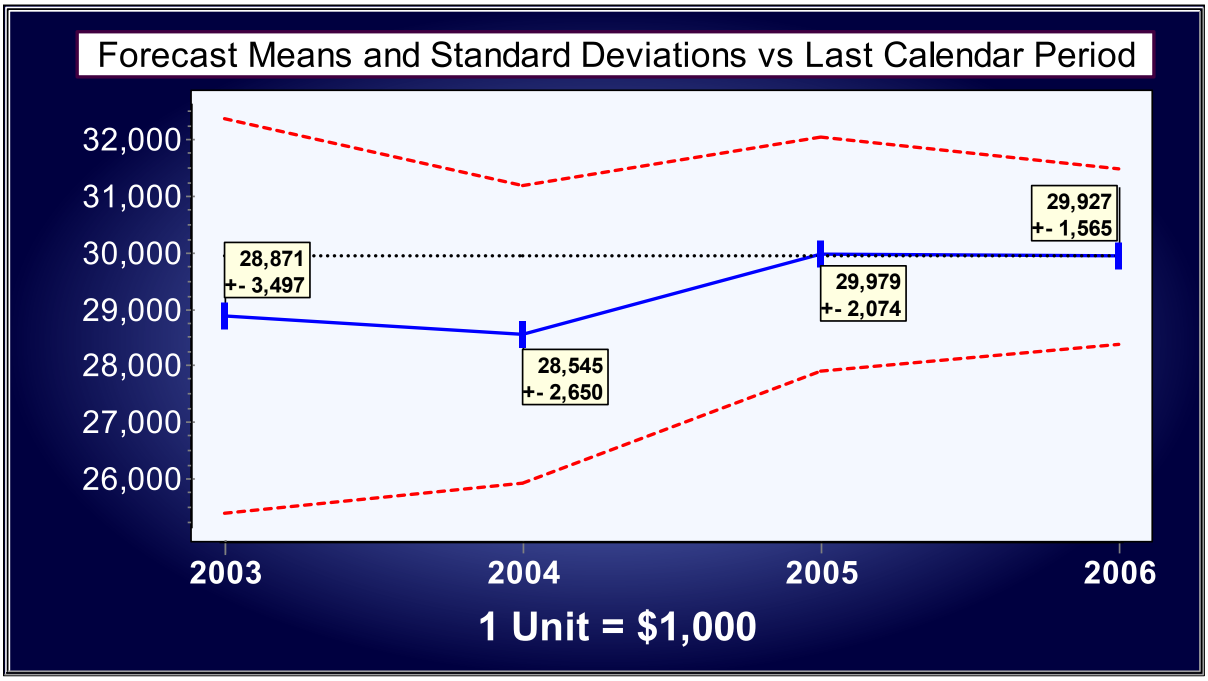
The development trends are as could be expected from the previous display – a sharp increase followed by a second slight increase, flat, then a steady decline. Two accident level increases have been identified. The first development and later development periods have higher variability than the middle periods – again expected since the more paid the less variation on a percentage scale.



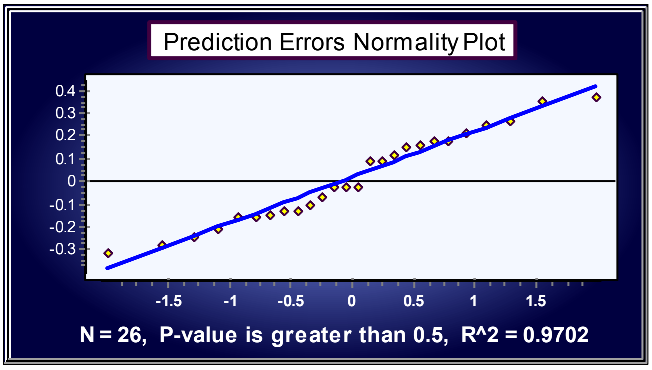


The normality of the residuals is very good; P-value exceeds 0.5.

Forecasting with the identified PTF model and validation analyses



The estimates of the total reserve are stable as we go back in time, though there seems to be an increase in 2005 and 2006. This increase, however, is still within the bounds of the standard deviations of the previous calculations. This is primarily a result of the accident levels being estimated more accurately as more data are available.



Statistically, we are doing a good job of predicting the validation data conditional on the model structure and exposure. The residuals of the projections are normally distributed.

For this dataset, the forecast was extended by five development periods. The same interpretation of the colours applies as with the previous table. All projected cells have a log-normal distribution since we take the inverse of the normal distributions fitted to the log of the dollars paid.

Accident Period vs Development Period

	Cal. Per. Total	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Reserve	Ultimate
1997	242	242	963	1,182	1,182	844	602	438	313	223	160	114	82	58	42	30	326	6,752
	255	255	1,036	1,028	1,180	815	759	573	353	313	185	30	22	16	12	9	60	60
1998	1,205	242	963	1,182	1,182	844	602	438	313	223	160	114	82	58	42	30	486	5,978
	1,344	308	968	1,254	971	677	524	273	349	168	40	30	22	16	12	9	81	81
1999	2,418	272	1,083	1,330	1,330	949	677	492	352	251	180	128	92	66	47	34	798	7,459
	2,299	303	1,210	1,334	1,217	1,101	639	383	226	62	46	34	25	19	14	10	126	126
2000	3,730	272	1,082	1,330	1,330	949	677	492	352	251	180	128	92	66	47	34	1,150	7,071
	3,931	287	1,018	1,437	1,142	864	727	446	85	62	46	34	25	19	14	10	166	166
2001	4,783	344	1,370	1,683	1,683	1,201	857	623	445	318	227	163	116	83	60	43	2,079	9,167
	4,577	339	1,147	1,578	2,023	1,338	871	149	108	79	59	43	32	24	18	13	285	285
2002	5,820	344	1,370	1,682	1,682	1,201	857	623	445	318	227	163	116	83	60	43	2,936	8,837
	5,556	289	1,165	1,533	1,618	1,356	119	148	108	79	59	43	32	24	18	13	328	328
2003	6,715	344	1,370	1,682	1,682	1,201	857	623	445	318	227	163	116	83	60	43	4,136	9,605
	6,519	303	1,414	1,985	1,565	157	114	148	108	79	59	43	32	24	18	13	382	382
2004	7,455	344	1,370	1,682	1,682	1,201	857	623	445	318	227	163	116	83	60	43	5,819	9,250
	7,347	348	1,530	1,733	220	157	114	148	108	79	59	43	32	24	18	13	454	454
2005	7,985	344	1,370	1,682	1,682	1,201	857	623	445	318	227	163	116	83	60	43	7,501	9,072
	8,231	158	1,413	220	220	157	114	148	108	79	59	43	32	24	18	13	524	524
2006	8,364	344	1,370	1,682	1,682	1,201	857	623	445	318	227	163	116	83	60	43	8,872	9,143
	8,282	271	184	220	220	157	114	148	108	79	59	43	32	24	18	13	567	567
	Total Fitted/Paid		2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	Total Reserve	Total Ultimate
Cal. Per. Total	48,707		8,293	7,117	5,575	3,992	2,863	2,036	1,434	1,001	692	465	302	186	102	43	34,102	82,433
	48,221		510	483	423	360	315	277	214	164	124	90	64	43	27	13	2,038	2,038

1 Unit = $1,000

The right corner of the table above resized below for readability.

18	13	454	454
60	43	7,501	9,072
18	13	524	524
60	43	8,872	9,143
18	13	567	567
2019	2020	Total Reserve	Total Ultimate
102	43	34,102	82,433
27	13	2,038	2,038

Units are in 000s

The total projected reserve at year end 2006 has a mean of 34,102,000 with a standard deviation: 2,038,000.

How do we know if prior year estimates of ultimates are consistent on updating?

The two columns on the right provide some statistics conditional on the next calendar year's (2007) data (not observed yet).

If on updating the model using 2007 data there is no significant change in calendar year (zero) trend and no significant change in decay trend from period 3-9, then the mean ultimate of all the mean conditional on 2007 data ultimates, is the mean ultimate as at end 2006.

The second column from the right represents on average the SD of ultimate given 2007 data (note reduction due decrease in parameter uncertainty with more data and forecasting horizon not as far), whereas the column on extreme right represents the SD of the conditional on 2007 expectation of mean ultimate. It gives an idea of possible statistical variation in mean ultimate that maintains consistent estimates of prior ultimates on update

There is expected to be little change in the re estimation of prior year ultimates assuming that the trend applied in the forecast holds true for the next calendar year(s).

<table>
<tr><th colspan="8">Accident Yr Summary</th></tr>
<tr><th rowspan="2">Acc. Yr</th><th colspan="2">Mean</th><th>Standard</th><th colspan="2">CV</th><th colspan="2">Cond. on Next Cal. Per.</th></tr>
<tr><th>Outstanding</th><th>Ultimate</th><th>Dev.</th><th>Outstanding</th><th>Ultimate</th><th>Std.Dev.|Data</th><th>+-Ult|Data</th></tr>
<tr><td>1997</td><td>326</td><td>6,752</td><td>60</td><td>0.18</td><td>0.01</td><td>36</td><td>47</td></tr>
<tr><td>1998</td><td>486</td><td>5,978</td><td>81</td><td>0.17</td><td>0.01</td><td>51</td><td>63</td></tr>
<tr><td>1999</td><td>798</td><td>7,459</td><td>126</td><td>0.16</td><td>0.02</td><td>82</td><td>96</td></tr>
<tr><td>2000</td><td>1,150</td><td>7,071</td><td>166</td><td>0.14</td><td>0.02</td><td>111</td><td>124</td></tr>
<tr><td>2001</td><td>2,079</td><td>9,167</td><td>285</td><td>0.14</td><td>0.03</td><td>192</td><td>210</td></tr>
<tr><td>2002</td><td>2,936</td><td>8,837</td><td>328</td><td>0.11</td><td>0.04</td><td>255</td><td>206</td></tr>
<tr><td>2003</td><td>4,136</td><td>9,605</td><td>382</td><td>0.09</td><td>0.04</td><td>293</td><td>246</td></tr>
<tr><td>2004</td><td>5,819</td><td>9,350</td><td>454</td><td>0.08</td><td>0.05</td><td>344</td><td>296</td></tr>
<tr><td>2005</td><td>7,501</td><td>9,072</td><td>524</td><td>0.07</td><td>0.06</td><td>417</td><td>317</td></tr>
<tr><td>2006</td><td>8,872</td><td>9,143</td><td>567</td><td>0.06</td><td>0.06</td><td>485</td><td>293</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Total</td><td>34,102</td><td>82,433</td><td>2,038</td><td>0.06</td><td>0.02</td><td>1,443</td><td>1,439</td></tr>
<tr><td colspan="8">1 Unit = $1,000</td></tr>
</table>

From this table, we can deduce that expected change in ultimate given the next years data is 1,439,000. The expected new standard deviation of the reserve distribution is expected to drop to 1,443,000.

The reserve calculations are more variable than the previous dataset, but, as the validation indicated previously, the reserves are expected to be stable – again assuming that the trend assumptions for the future match the trend assumptions assumed in the forecast scenarios.

Comparison of Last Cal. Yr with First Three Future Cal. Yrs				
Excl. Dev. Year	Last Calendar Year (2006)	Calendar Year	Forecast	
			Mean	Std dev
0:0	8,011	2007	8,293	510
0:1	6,598	2008	7,117	483
0:2	4,865	2009	5,575	423
1 Unit = $1,000				

2003	6,715	344	1,370	1,682	1,682	1,201
	6,519	505	1,414	1,985	1,565	157
2004	7,455	344	1,370	1,682	1,682	1,201
	7,347	248	1,550	1,733	220	157
2005	7,985	344	1,370	1,682	1,682	1,201
	8,221	158	1,413	220	220	157
2006	8,364	344	1,370	1,682	1,682	1,201
	8,282	271	184	220	220	157
	Total Fitted/Paid		2007	2008	2009	2010
Cal. Per. Total	48,707		8,293	7,117	5,575	3,992
	48,331		510	483	423	360

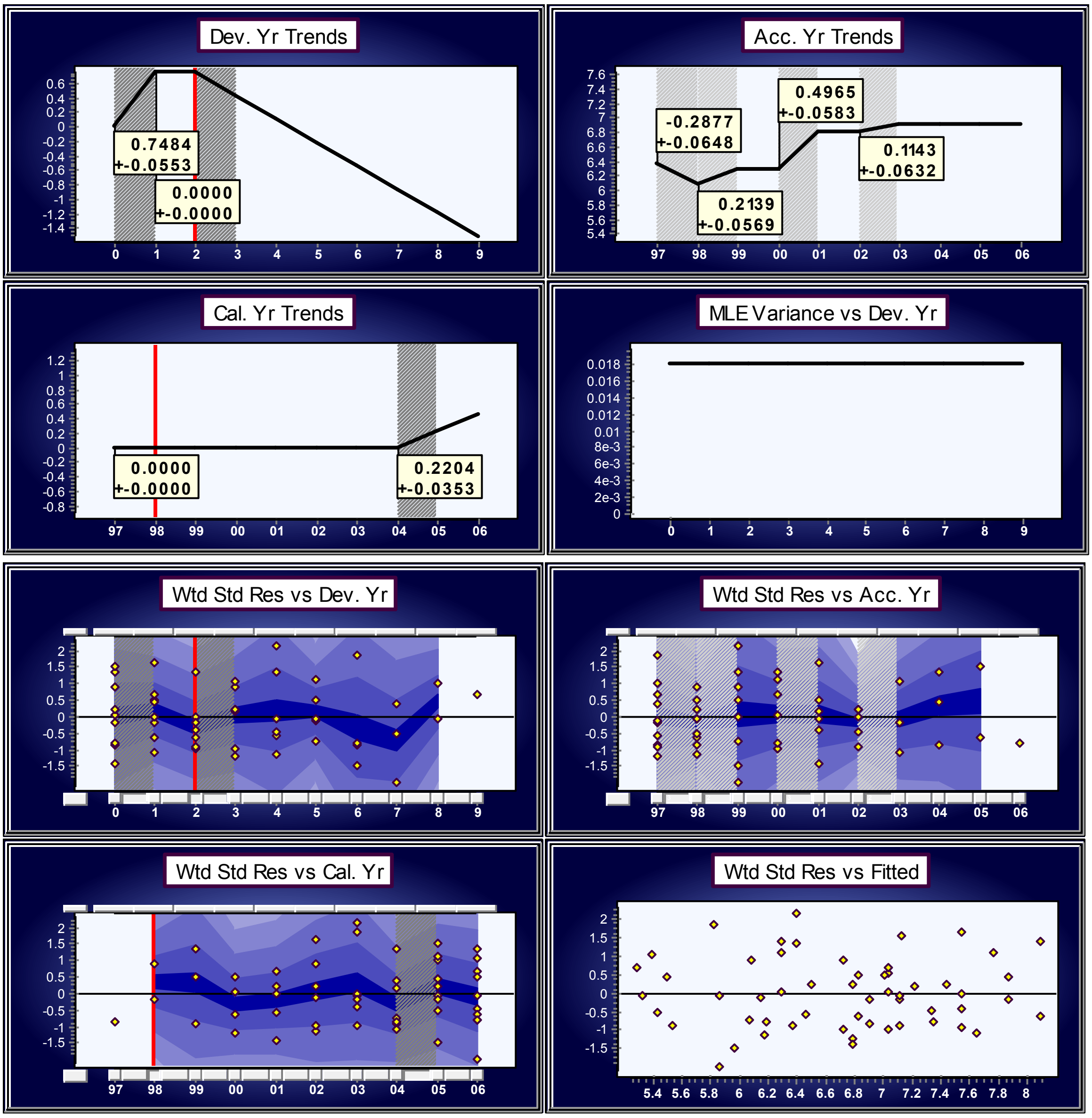
The liability stream is consistent for the next calendar years compared to the previously paid amounts.

Incurred Losses							
Acc. Yr	Premium	Paid To 2006	Incurred To 2006	CRE 2006	Mean Reserve	Mean Ultimate	Standard Dev.
1997	6,891	6,426	6,644	218	326	6,752	60
1998	7,525	5,492	5,694	202	486	5,978	81
1999	8,609	6,661	6,924	263	798	7,459	126
2000	9,318	5,921	6,354	433	1,150	7,071	166
2001	9,212	7,088	8,272	1,184	2,079	9,167	285
2002	8,843	5,901	7,328	1,427	2,936	8,837	328
2003	8,456	5,469	8,233	2,764	4,136	9,605	382
2004	7,984	3,531	7,522	3,991	5,819	9,350	454
2005	8,446	1,571	4,567	2,996	7,501	9,072	524
2006	9,802	271	1,652	1,381	8,872	9,143	567
Total	85,086	48,331	63,190	14,859	34,102	82,433	2,038
1 Unit = $1,000							

As before, this summary table above includes both the premium, incurred to date, and CRE.

Case Reserve Estimates (CREs)

The case reserve estimates were modelled for comparison with the Paid Losses. Are the same trends being measured?



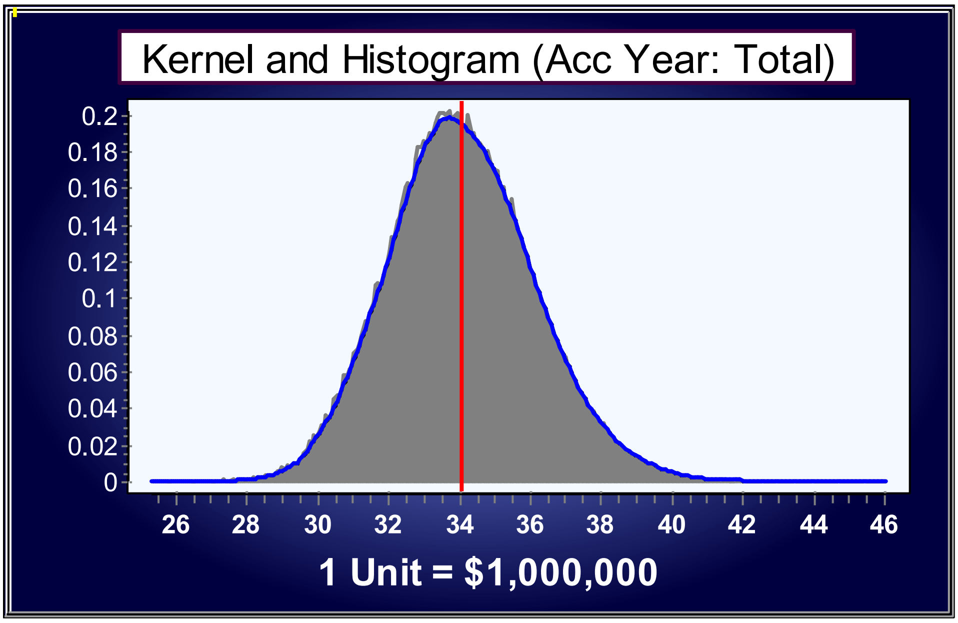
The Case Reserve Estimates for this dataset show significantly more movement than the Paid Losses. They increase in 2002-2003 and have a very strong calendar trend from 2004 onwards as though it was realised that the case reserve estimates were too low and there has been an effort made to ramp them up. The Case Reserves Estimates team would need to be contacted to confirm this.

It should be noted that the case reserve estimates are significantly less the projected total amount to be paid.

Reserve probability distributions by calendar year, accident year and total

The PTF framework also greatly facilitates detailed projection of the distribution of future payments for risk capital calculations and the breakdown of payments by calendar year for asset-liability matching.

It should be noted for the calculation of the sum of log-normal distributions there is no closed analytical form – simulation is the only means to gain a description of the aggregate distribution (total reserve in this case).



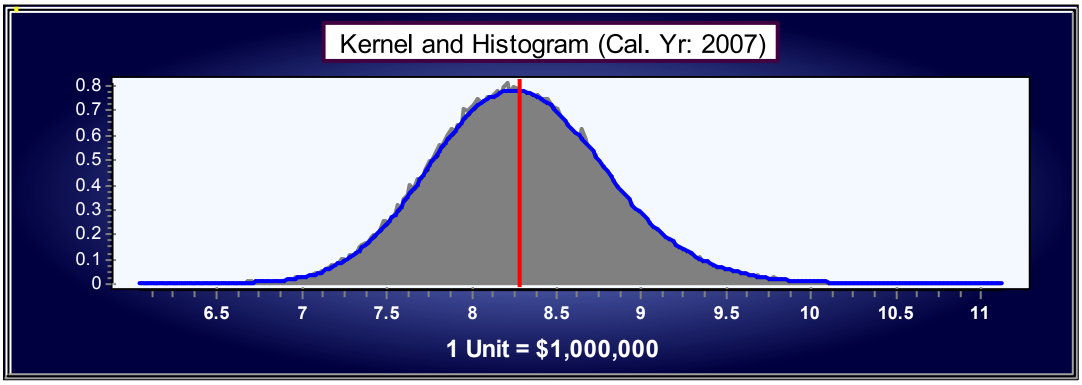
For this scenario, the total reserve distribution is only mildly skewed.

Quantile Statistics, VaR and T-VaR (Acc Year: Total)

%	Sample				Kernel			
	Quantile	# S.D.'s	VaR	T-VaR	Quantile	# S.D.'s	VaR	T-VaR
99.5	39.828	2.810	5.726	6.503	39.863	2.827	5.761	6.536
99.4	39.667	2.731	5.565	6.359	39.708	2.751	5.606	6.396
99.3	39.535	2.666	5.433	6.236	39.576	2.686	5.474	6.278
99.2	39.432	2.616	5.330	6.129	39.460	2.629	5.358	6.155
99.1	39.319	2.560	5.217	6.034	39.356	2.578	5.254	6.067
99.0	39.235	2.519	5.133	5.948	39.263	2.532	5.161	5.975
98.0	38.578	2.196	4.476	5.361	38.602	2.208	4.500	5.384
97.0	38.147	1.985	4.045	4.990	38.177	2.000	4.075	5.016
96.0	37.846	1.837	3.744	4.715	37.863	1.846	3.761	4.732
95.0	37.583	1.708	3.481	4.494	37.608	1.720	3.506	4.513
94.0	37.372	1.605	3.270	4.307	37.394	1.615	3.292	4.325
93.0	37.189	1.515	3.088	4.145	37.210	1.525	3.109	4.164
92.0	37.032	1.438	2.930	4.003	37.048	1.446	2.946	4.018
91.0	36.886	1.366	2.784	3.875	36.901	1.374	2.799	3.889
90.0	36.751	1.300	2.649	3.759	36.766	1.307	2.664	3.774
89.0	36.626	1.239	2.524	3.653	36.641	1.246	2.539	3.668
88.0	36.505	1.179	2.403	3.553	36.524	1.188	2.422	3.570

Mean = 34.102, S.D. = 2.038, Provision = 34.102, 1 Unit = $1,000,000

The Sample columns refer to the sample of 100,000 draws from the joint distribution of forecast cells, the Kernel columns refer to a smoothed version of the empirical distribution. The VaR and T-VaR figures are based on reserving at the mean; they can be recomputed by a shift if a different figure is used for the reserve.

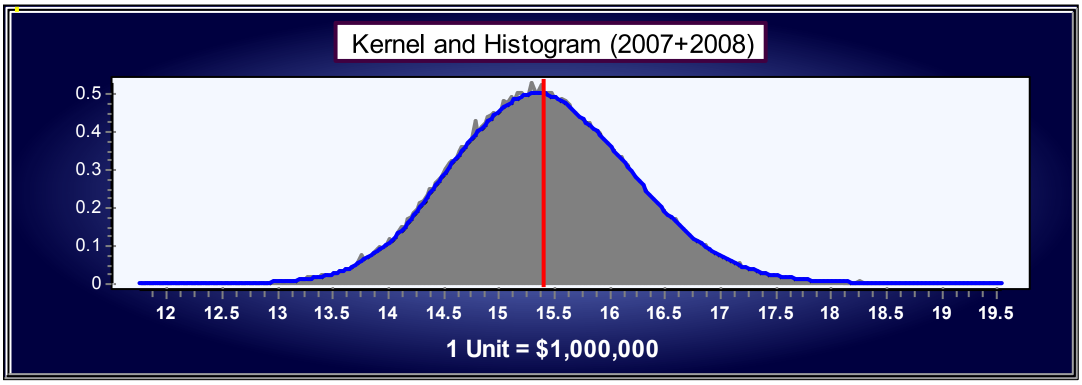


Quantile Statistics, VaR and T-VaR (Cal. Yr: 2007)

%	Sample				Kernel			
	Quantile	# S.D.'s	VaR	T-VaR	Quantile	# S.D.'s	VaR	T-VaR
99.5	9.711	2.780	1.419	1.619	9.722	2.800	1.429	1.628
99.4	9.677	2.712	1.384	1.582	9.684	2.726	1.391	1.592
99.3	9.643	2.647	1.351	1.552	9.650	2.660	1.358	1.557
99.2	9.614	2.589	1.321	1.525	9.621	2.602	1.328	1.532
99.1	9.584	2.530	1.291	1.500	9.594	2.550	1.301	1.507
99.0	9.560	2.484	1.268	1.478	9.570	2.502	1.277	1.486
98.0	9.396	2.162	1.103	1.328	9.404	2.178	1.111	1.336
97.0	9.298	1.969	1.005	1.236	9.302	1.979	1.010	1.241
96.0	9.220	1.817	0.927	1.168	9.226	1.829	0.933	1.174
95.0	9.158	1.696	0.866	1.113	9.164	1.708	0.872	1.119
94.0	9.108	1.597	0.815	1.068	9.112	1.606	0.820	1.072
93.0	9.062	1.508	0.770	1.028	9.067	1.518	0.775	1.033
92.0	9.022	1.428	0.729	0.993	9.027	1.440	0.735	0.998
91.0	8.987	1.360	0.694	0.962	8.991	1.369	0.699	0.966
90.0	8.955	1.297	0.662	0.934	8.958	1.304	0.666	0.938
89.0	8.924	1.237	0.631	0.908	8.928	1.245	0.635	0.912
88.0	8.895	1.180	0.602	0.883	8.900	1.190	0.607	0.888

Mean = 8.293, S.D. = 0.510, Provision = 8.293, 1 Unit = $1,000,000

The distribution of the next calendar year, reserve risk, is fairly symmetric but as with dataset D, a wide range of possible values simulated ranging from 5M to 11M.



The volatility in the first two calendar years is high, but the distribution itself is quite symmetric with the sum of the two years (from the simulations) ranging from just under 12M to around 20M.

%	Sample Quantile	Sample # S.D.'s	Sample VaR	Sample T-VaR	Kernel Quantile	Kernel # S.D.'s	Kernel VaR	Kernel T-VaR
99.5	17.596	2.735	2.186	2.463	17.605	2.746	2.195	2.472
99.4	17.537	2.661	2.127	2.411	17.546	2.673	2.136	2.420
99.3	17.475	2.584	2.065	2.366	17.496	2.610	2.086	2.383
99.2	17.433	2.530	2.023	2.326	17.452	2.555	2.042	2.344
99.1	17.399	2.488	1.988	2.291	17.414	2.507	2.004	2.304
99.0	17.366	2.447	1.956	2.259	17.380	2.464	1.970	2.271
98.0	17.126	2.147	1.716	2.043	17.139	2.163	1.729	2.054
97.0	16.972	1.954	1.562	1.908	16.982	1.966	1.571	1.917
96.0	16.851	1.803	1.441	1.806	16.863	1.818	1.453	1.817
95.0	16.763	1.692	1.353	1.724	16.770	1.701	1.360	1.730
94.0	16.685	1.595	1.275	1.655	16.691	1.603	1.281	1.661
93.0	16.613	1.505	1.203	1.595	16.622	1.517	1.212	1.604
92.0	16.554	1.431	1.144	1.543	16.561	1.440	1.151	1.548
91.0	16.499	1.362	1.089	1.495	16.505	1.370	1.095	1.500
90.0	16.447	1.297	1.037	1.452	16.453	1.305	1.043	1.457
89.0	16.399	1.238	0.989	1.412	16.406	1.246	0.996	1.417
88.0	16.354	1.180	0.944	1.375	16.361	1.190	0.951	1.382

Quantile Statistics, VaR and T-VaR (2007+2008)

Mean = 15.410, Sample S.D. = 0.799, Provision = 15.410, 1 Unit = $1,000,000

In ICRFS-Plus, using a model from the Probabilistic Trend Family (PTF), it is feasible to project future underwriting years as well as reserve periods. In order to facilitate this, it is assumed that there are no material premium changes for 2007 and 2008 (the relative exposure is comparable to 2006). Therefore, the level and exposure changes can be maintained for these two periods.

Accident Yr Summary

Acc. Yr	Mean Outstanding	Mean Ultimate	Standard Dev.	CV Outstanding	CV Ultimate	Cond. on Next Cal. Per. Std.Dev.|Data	Cond. on Next Cal. Per. +-Ult|Data
2007	9,587	9,587	629	0.07	0.07	551	303
2008	9,594	9,594	630	0.07	0.07	560	289
Total	19,180	19,180	1,072	0.06	0.06	897	587

1 Unit = $1,000

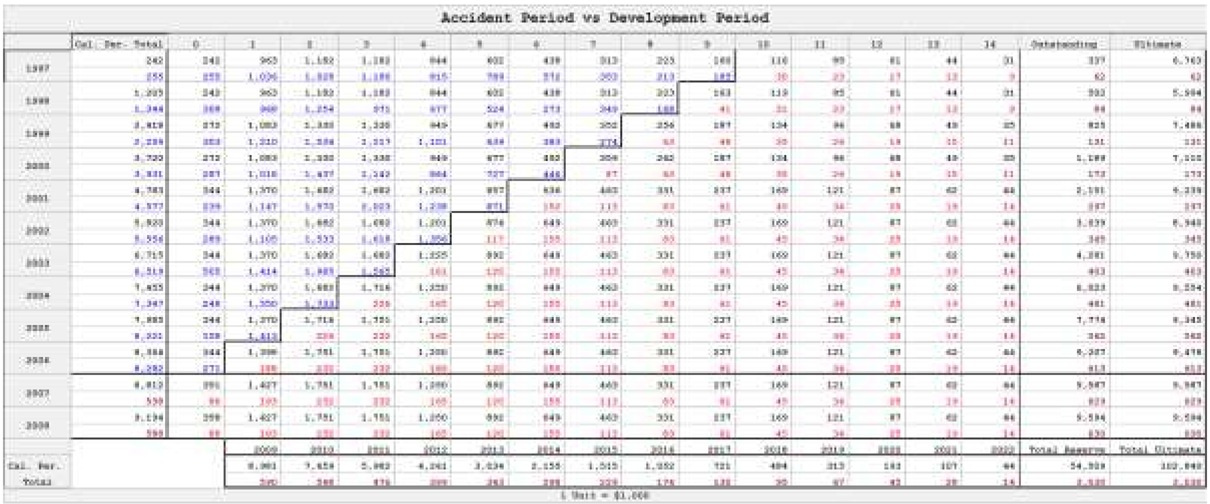
The above table shows the projected ultimates for the future underwriting years along with the conditional information on how the ultimates are expected to change given the next years data. Again, the additional information in 2007 is expected to reduce the volatility in the ultimate since we will then know the 2007 figures exactly.

Calendar Yr Summary				
Calendar Yr	Mean Outstanding	Standard Dev.	CV Outstanding	Cum. Payment as % of total
2007	351	86	0.25	1.83
2008	1,785	214	0.12	11.13
2009	3,178	309	0.10	27.70
2010	3,503	346	0.10	45.97
2011	3,001	298	0.10	61.61
2012	2,142	215	0.10	72.78
2013	1,541	205	0.13	80.82
2014	1,112	199	0.18	86.61
2015	795	148	0.19	90.76
2016	568	111	0.20	93.72
2017	406	84	0.21	95.84
2018	291	63	0.22	97.35
2019	208	48	0.23	98.44
2020	149	36	0.24	99.21
2021	107	28	0.26	99.77
2022	44	14	0.31	100.00
Total	19,180	1,072	0.06	100.00

1 Unit = $1,000

The above table illustrates the expected liability stream pattern for the future two underwriting years 2007 and 2008 only.

A combined reserve and underwriting table is then computed and is displayed below.



Summary by calendar year required for cost of capital calculations

Of primary interest is the projection of the next two calendar years. Within the reserving context, the projections can be taken directly from the forecast table.

Calendar Yr Summary

Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve	Cum. Payment as % of total
2007	8,293	510	0.06	24.32
2008	7,117	483	0.07	45.19
2009	5,575	423	0.08	61.54
2010	3,992	360	0.09	73.24
2011	2,863	315	0.11	81.64
2012	2,036	277	0.14	87.61
2013	1,434	214	0.15	91.81
2014	1,001	164	0.16	94.75
2015	692	124	0.18	96.78
2016	465	90	0.19	98.14
2017	302	64	0.21	99.03
2018	186	43	0.23	99.57
2019	102	27	0.26	99.87
2020	43	13	0.31	100.00

1 Unit = $1,000

However, this projection assumes that no further business is written for this line and so it forecasts the cash flow for the reserving risk. The expected means in the next two years, in the triangle without projecting further underwriting years, are 8,293,000 +- 510,000 and 7,117,000 +- 483,000 respectively. Note that the distribution is not symmetric and the +- component refers to the standard error of the distribution.

The liability stream is reflective of the development pattern and four years into the future 75% of the liabilities have been paid. Four years into the projection the previously analysed dataset had already paid 97% of the total liabilities. As a result, any calendar year period changes will have more of an effect on this dataset.

As before, the future two accident periods are projected and the combined table of reserve and underwriting risk produced. For projecting the combined risk, a conservative assumption was applied to the next two calendar periods of a 2% +- 1% trend. The 2% trend was taken from the calendar trend 2004-2006 trend of 2.66% +- 3.64% which is not significant and therefore not retained in the optimal model fitted above. In order to be conservative given the sensitivity of this data to calendar shifts, the 2%+-1% trend was applied.

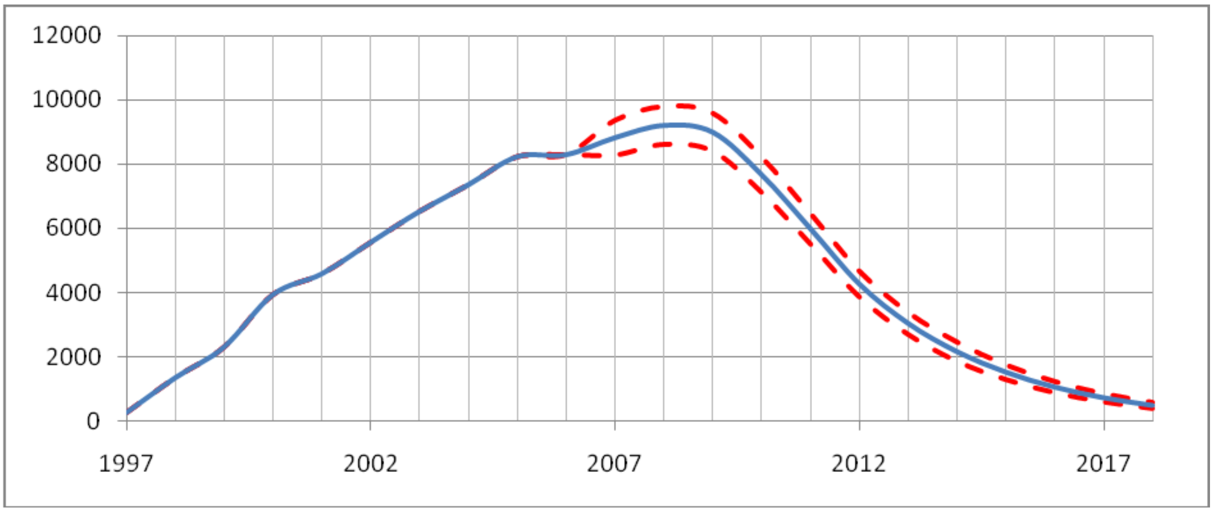
This assumption of 2% +- 1% for the next two years seems in line with the paid to date by calendar year and should be included.

Calendar Yr Summary

Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve	Cum. Payment as % of total
2007	8,812	538	0.06	16.17
2008	9,194	588	0.06	33.03
2009	8,981	590	0.07	49.51
2010	7,659	548	0.07	63.56
2011	5,982	476	0.08	74.53
2012	4,261	399	0.09	82.35
2013	3,034	343	0.11	87.92
2014	2,155	298	0.14	91.87
2015	1,515	229	0.15	94.65
2016	1,052	174	0.17	96.58
2017	721	130	0.18	97.90
2018	484	95	0.20	98.79
2019	315	67	0.21	99.37
2020	193	45	0.23	99.72
2021	107	28	0.26	99.92
2022	44	14	0.31	100.00
Total	54,509	3,030	0.06	100.00

1 Unit = $1,000

The projected means in 2007 and 2008 are now 8,812,000 and 9,914,000 respectively and include the projection for the first development period.



A comparison with the paid to date by calendar period is made above with the projected calendar year means incorporating the next two underwriting periods. The dashed red line represents the mean +- one standard deviation. The projections are consistent with the paid-to-date thus far and are assuming no changes in exposure or level changes. An additional 2% +- 1% calendar trend was applied for 2007 and 2008 in order to be consistent and conservative with the previously observed payments.

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Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev.|Data	+-Ult|Data
1997	337	6,763	62	0.18	0.01	38	49
1998	502	5,994	84	0.17	0.01	54	65
1999	825	7,486	131	0.16	0.02	85	100
2000	1,189	7,110	173	0.15	0.02	115	129
2001	2,151	9,239	297	0.14	0.03	200	219
2002	3,039	8,940	345	0.11	0.04	266	219
2003	4,281	9,750	403	0.09	0.04	305	264
2004	6,023	9,554	481	0.08	0.05	359	321
2005	7,774	9,345	560	0.07	0.06	435	353
2006	9,207	9,478	613	0.07	0.06	507	344
2007	9,587	9,587	629	0.07	0.07	551	303
2008	9,594	9,594	630	0.07	0.07	560	289
Total	54,509	102,840	3,030	0.06	0.03	2,120	2,166

Accident Yr Summary

1 Unit = $1,000

The table above is consistent with the previous summary, but includes the additional two rows for the expected figures of the underwriting risk in 2007 and 2008. In the PTF modelling framework, the projection of reserving and underwriting risk are not seen as two separate problems but rather the two risks can (and should) be treated as liability risk. As mentioned previously, the 2%+-1% future calendar trend is added.

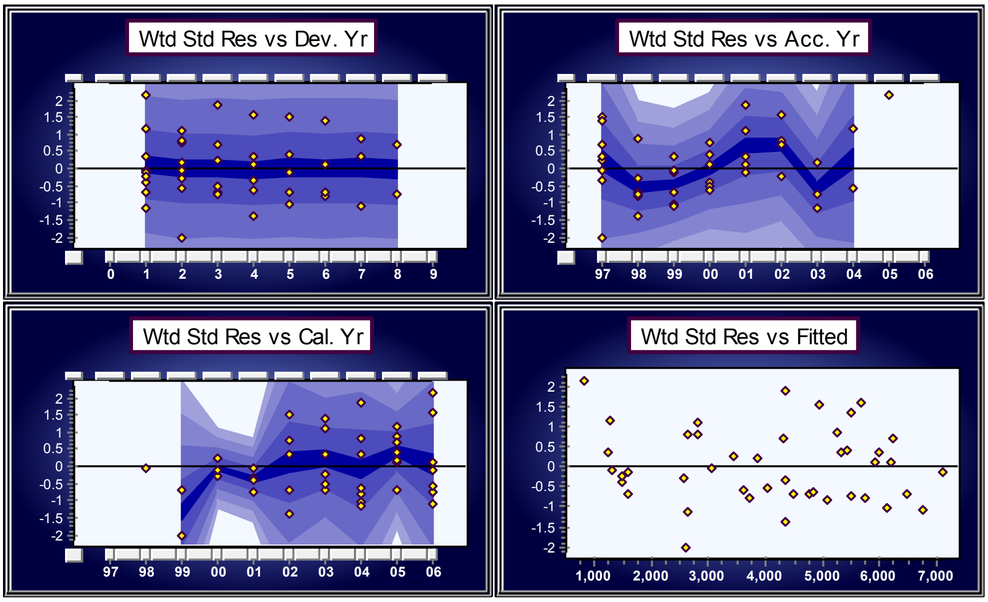
As before, the expected change in ultimate for each year and total as the next years data becomes available is also quantified.

The Mack Method

The Mack Method was applied to the Paid Losses and subsequently to the Incurred Loss array.

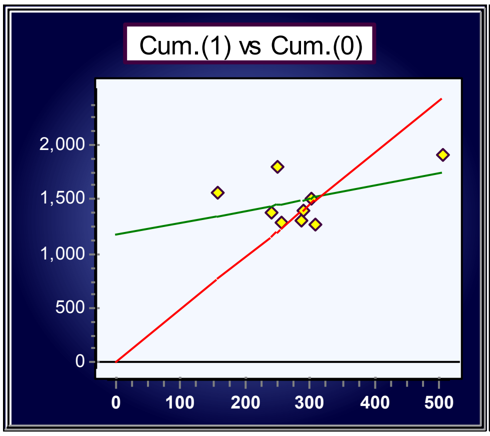
Mack Method applied to Paid Loss data

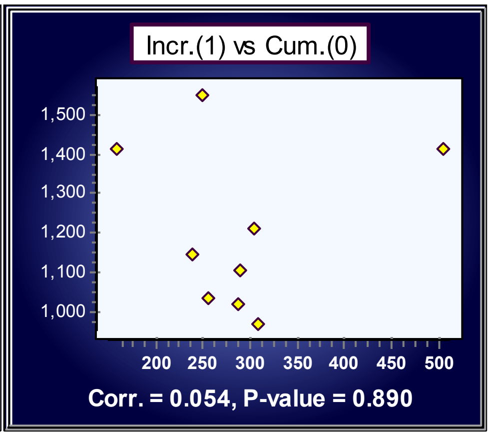
As with the previous dataset, the same set of diagnostics are produced for the Mack Method on the Paid Losses. First, the residuals versus the three directions and the fitted values are examined. The other assumptions of the Mack method, denoted previously are then tested.



The residual displays indicate underfitting is occurring, particularly by calendar year and there are a few accident years that are underfitted. The trends in the method are less than the trends in the data.

However, the next step is to test for the necessity of an intercept and whether the ratios from the previous cumulative are indicators of the next incremental.





The displays are obviously distorted by the extreme observations, but even without those observations it is clear that an intercept would provide a better alternative. There is no relationship between the incrementals at period 1 versus the cumulative at period 0 (right hand plot).

Since the residuals indicate problems, however, the projections for the method are produced but without further discussion.

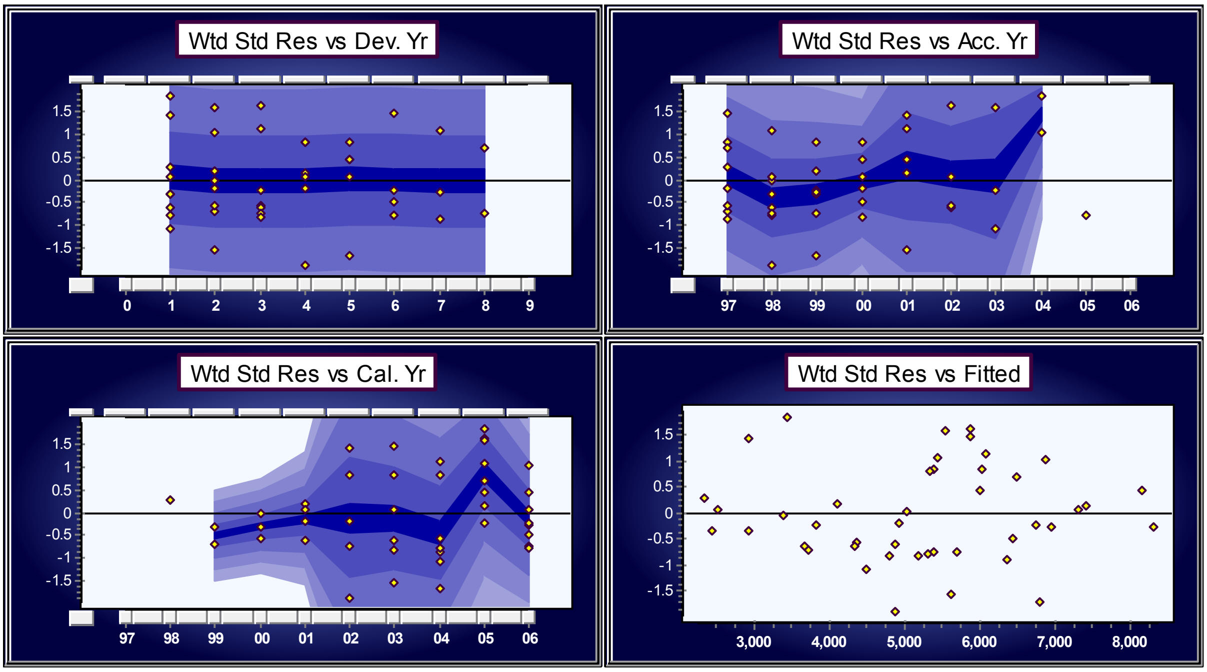
Incremental Forecasts

	0	1	2	3	4	5	6	7	8	9	Outstanding	Ultimate
1997	255	1,069	1,312	1,103	858	617	401	325	202	185	0	6,426
	255	1,036	1,028	1,180	815	789	572	353	213	185	0	0
1998	308	1,291	1,297	1,203	859	598	370	285	179	163	163	5,655
	308	968	1,254	971	677	524	273	349	168	3	3	3
1999	303	1,270	1,537	1,449	1,046	767	472	366	224	204	428	7,089
	303	1,210	1,534	1,217	1,101	639	383	274	21	3	22	22
2000	287	1,203	1,326	1,304	953	679	431	339	210	192	741	6,662
	287	1,018	1,437	1,142	864	727	446	95	20	4	104	104
2001	239	1,001	1,408	1,406	1,221	889	557	438	271	248	1,514	8,602
	239	1,147	1,570	2,023	1,238	871	169	113	25	7	226	226
2002	289	1,211	1,417	1,392	1,115	844	530	417	258	236	2,285	8,186
	289	1,105	1,533	1,618	1,356	147	164	110	24	9	282	282
2003	505	2,116	1,950	1,857	1,341	974	612	481	298	272	3,979	9,448
	505	1,414	1,985	1,565	185	162	180	121	28	12	401	401
2004	248	1,039	1,827	1,680	1,278	928	583	458	284	259	5,470	9,001
	248	1,550	1,733	389	203	172	180	122	35	23	777	777
2005	158	662	1,596	1,507	1,146	832	523	411	255	232	6,503	8,074
	158	1,413	165	374	200	167	171	116	35	24	842	842
2006	271	1,136	1,429	1,349	1,026	745	468	368	228	208	6,958	7,229
	271	482	513	580	399	299	226	166	85	75	2,598	2,598
		2007	2008	2009	2010	2011	2012	2013	2014	2015	Total Outstanding	Total Ultimate
Cal. Yr Totals	****	7,880	6,571	4,915	3,429	2,260	1,435	882	460	208	28,041	76,372
	****	****	****	****	****	****	****	****	****	****	3,067	3,067

1 Unit = $1,000

Instead we move onto examining the projections of the outstanding based on the Incurred Losses. The effect of the case reserve estimates increase may distort the calculation of the outstanding.

Mack Method applied to Incurred Loss data



The Incurred Losses shows the calendar increase effect of the sudden ramping up of the Case Estimates – 2005 in particular is high. The resulting forecast table is radically overstated (compared to PTF) given the Case Reserve Estimates still have to be allocated into the liability stream.

For the reserve component only, the forecast completing the square is shown below.

Incremental Forecasts

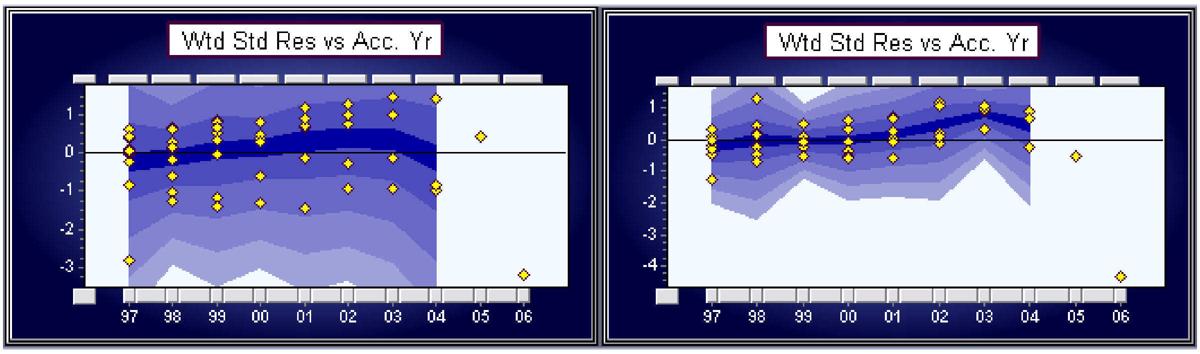
	0	1	2	3	4	5	6	7	8	9	Outstanding	Ultimate
1997	768	1,571	1,231	971	685	482	303	250	196	149	218	6,644
	768	1,724	909	844	660	656	554	171	209	149	0	0
1998	804	1,644	1,120	962	677	451	275	250	173	131	333	5,825
	804	1,464	1,103	820	397	460	149	337	160	4	4	4
1999	962	1,967	1,355	1,196	840	607	348	250	216	164	643	7,304
	962	1,781	1,447	1,011	978	214	307	224	25	4	26	26
2000	830	1,697	1,267	1,065	743	538	332	250	206	156	1,045	6,966
	830	1,735	1,168	870	869	634	248	104	24	5	112	112
2001	965	1,973	1,861	1,349	1,029	730	450	250	280	212	2,377	9,465
	965	2,802	959	1,647	1,055	844	244	117	31	8	287	287
2002	1,208	2,470	1,615	1,303	1,017	720	438	250	273	207	3,315	9,216
	1,208	2,061	1,297	1,730	1,032	282	240	116	32	11	423	423
2003	1,478	3,022	1,836	1,845	1,329	940	572	250	354	268	6,476	11,945
	1,478	2,238	2,749	1,768	233	333	286	130	39	14	573	573
2004	1,128	2,307	2,271	2,147	1,561	1,104	671	250	414	314	10,451	13,982
	1,128	3,469	2,925	372	264	371	319	139	46	19	825	825
2005	1,704	3,485	2,256	1,947	1,416	1,001	609	250	376	285	11,137	12,708
	1,704	2,863	687	402	287	363	305	133	57	34	1,474	1,474
2006	1,652	3,378	2,485	2,145	1,560	1,103	671	250	413	313	13,699	13,970
	1,652	822	832	552	397	426	343	139	90	61	2,729	2,729
		2007	2008	2009	2010	2011	2012	2013	2014	2015	Total Outstanding	Total Ultimate
Cal. Yr Totals	****	10,878	7,991	5,922	3,967	2,522	1,602	940	698	313	49,693	98,024
	****	****	****	****	****	****	****	****	****	****	3,702	3,702

1 Unit = $1,000

In conclusion, as with the previously analysed dataset, the optimal PTF model provides the best estimate of the total reserve. Furthermore, the optimal model is expected to be the most responsive to future calendar year changes given the measurement of calendar year trends is an inherent part of the model identification process in this framework.

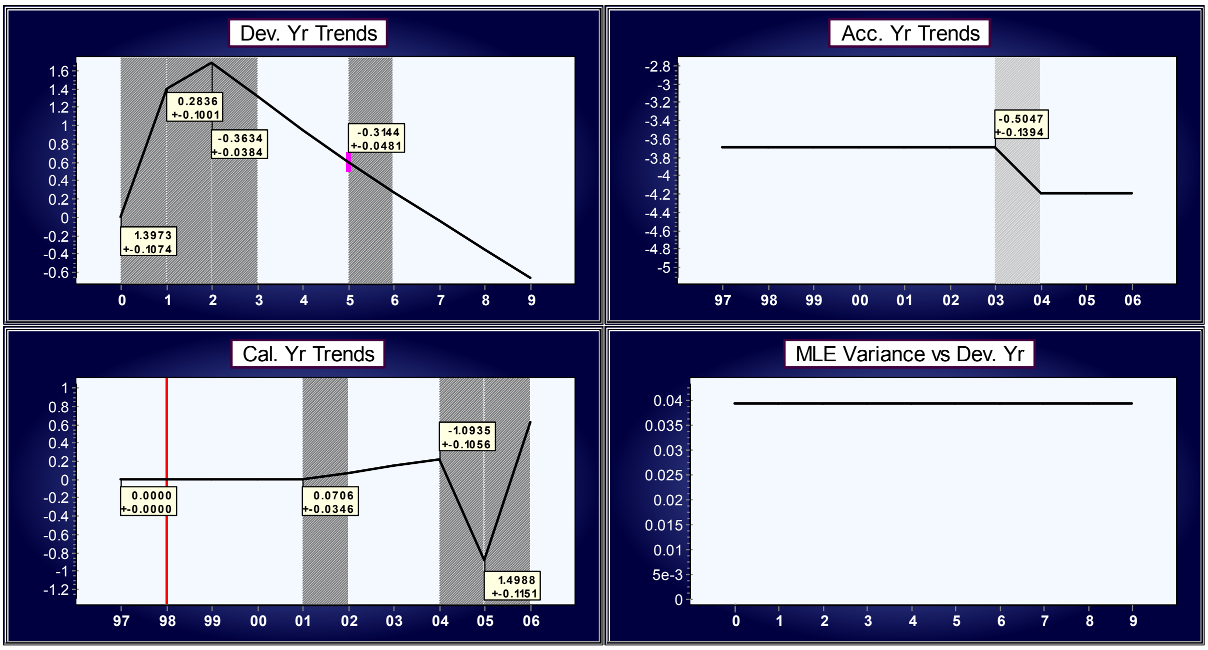
Data set H

Choice of Exposure Vector

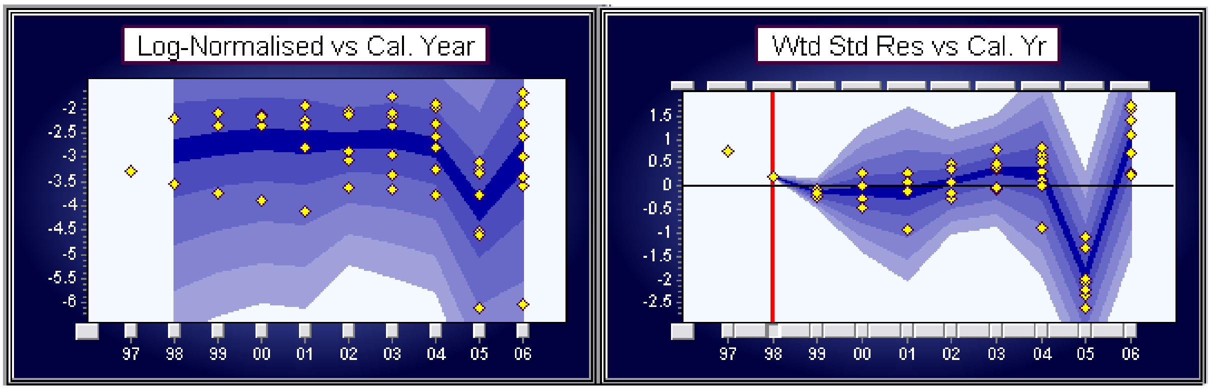


Comparison of Exposure vectors using the separation method indicates that Wages (left graph above) supplies the best exposure. The graph on the right uses "Exposure". Note the difference in scale.

Model Display

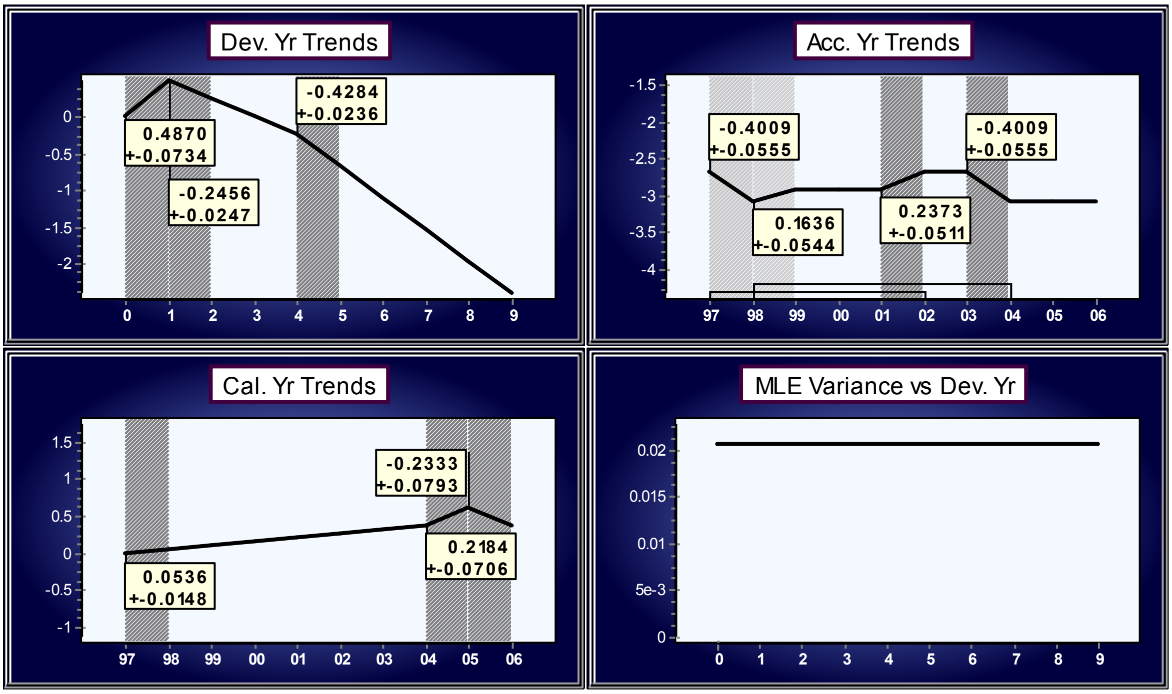


Modeling the trends in the three directions yields the above depicted model display. Note the very striking "kink" in the calendar year direction that occurs in 2005. This can be seen in the raw data as well as in a model display which adds only development and accident year parameters.

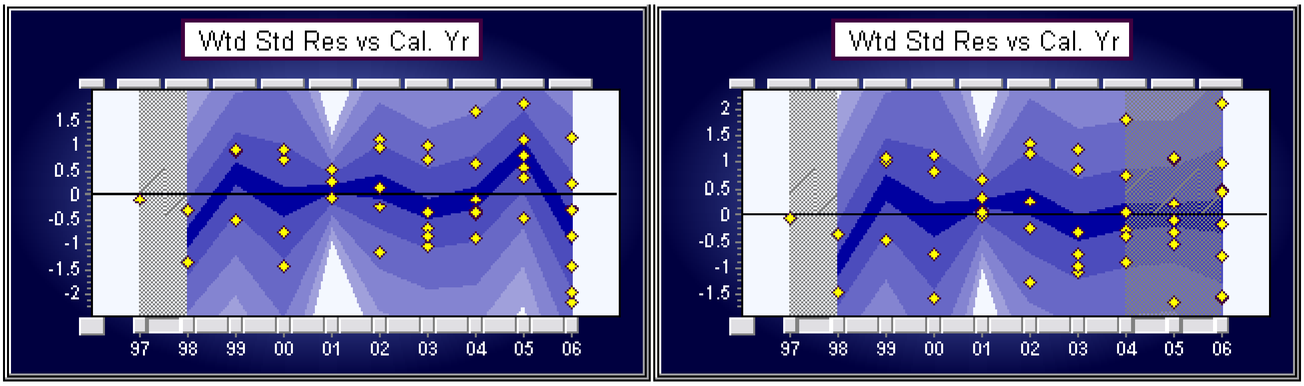


Left: The normalised (by Exposure) data plotted against calendar year on Log-scale. Right: The same data after development year and accident year trends have been adjusted for (estimated) only.

We can model the CREs to gain more insight into this calendar year trend changes. Here we see an opposite, although less pronounced effect in 2005.



CRE model display showing the trends in the three directions. Apart from a "kink" in 2005 there is a steady positive calendar (inflationary) trend of 0.0536+_0.0148. (Compare with the positive trend in the Paid Losses.)



Calendar Period plot of residuals for CRE model. Left: After development and accident trends have been accounted for. Right: After correction for 2005.

The indications are that the drop in Paid Losses experienced in 2005 with subsequent major increase across the range of accident years was not due to a change in economic conditions, but is quite likely to have been caused by a change in the claim clearing rate.

Fewer claims settled (closed) in 2000, hence more remaining open increasing the CREs by huge amount. Following year 2006 many more settled causing huge trend in paid losses but fewer remaining open reduce the CREs substantially.

We do not have the incremental number of claims closed (NCC) triangle available so we are unable to prove this hypothesis beyond all reasonable doubt.

Forecast Scenarios

We regard the 2005 "kink" as a transient claim-clearing-rate effect and look to the prior trends in the paid losses as a basis for our forecasting scenarios. We propose four and we shall extend them all to development year 30. We also believe that Scenario 1 is the most likely given the trend structure in the CREs relative to the paid losses.

The four are given in order of increasing conservatism. In practice the ability to update the models with new data as it becomes available means that we can monitor developments and eliminate the scenarios which have not eventuated. If reserving has been based on a conservative assumption which is not borne out then the difference between the old reserve figure and that derived from the revised scenario can be taken as profit.

In the past there was a zero calendar trend from 1997 to 2001 followed by a 0.0706+_0.0346 trend from 2001 to 2004.

Scenario 1: (-.19+_ the old)

The calendar year 2006 is approximately 26% higher than if the trend of 0.0706 +_ 0.0346 had continued from 2004 and 2006. By adopting a correction of -19% in 2007-2007 we rejoin the original trend in 2007.

	Iota	Std.Error
2006~2007	-0.1900	0.0346
2007~2036	0.0706	0.0346

The -19%+-3.46% is a correction for what we believe is an overshoot.

Scenario 2: The trend of 0.0706+_0.0346 recurs after two years of zero trend and persists for the remainder of the forecast period.

Scenario 3: The trend of 0.0706+_0.0346 recurs after one year of zero trend and persists for the remainder of the forecast period.

Scenario 4: The trend of 0.0706 +_ 0.0346 is maintained through the entire forecast period.

In all cases we assume the development decay is maintained for the entire period. Since this is a fairly sharp decay the effect of the positive calendar trend is minimal after a few years. It is also likely that as you add more development periods you will probably need to add decay parameters that are less sharp.

Forecast Results

The figure below shows the bottom left corner of the forecasting table for Scenario 1. The Numbers in the leftmost column show the calendar year totals for the past years with the observed values in blue and the model fitted values in black. We see that the fit is fairly close in all cases. The past portion of the forecast table is the same for all forecast scenarios.

In the bottom row of the table are the projected calendar year payments for the future years. The dark red numbers are the associated standard deviations.

1999	2,641	259	1,047	1,390	1,035	773	578	141	462	280	221	175
	2,428	246	1,155	1,440	1,191	955	624	103	340	66	55	52
2000	3,638	274	1,108	1,577	1,176	878	205	669	405	319	252	199
	3,510	221	1,020	1,374	1,227	810	248	542	92	79	70	62
2001	4,381	273	1,155	1,688	1,259	294	916	553	435	342	271	214
	4,313	176	1,275	1,835	1,076	409	1,043	127	105	90	80	71
2002	5,302	304	1,320	1,882	438	1,365	788	618	456	383	303	240
	5,164	299	1,386	1,540	400	1,649	190	151	126	109	96	85
2003	6,199	329	1,429	636	1,976	1,139	854	671	528	417	330	261
	6,561	286	1,670	497	2,059	269	221	178	149	128	112	99
2004	6,967	224	303	1,801	1,037	777	584	459	361	285	226	179
	6,606	266	273	2,166	288	207	171	139	116	99	86	75
2005	2,270	76	1,372	1,511	1,131	848	638	502	395	313	248	197
	2,062	0	904	379	297	241	200	163	136	116	100	87
2006	9,472	334	1,117	1,598	1,197	899	677	533	420	333	264	210
	9,373	25	277	422	335	274	225	186	156	132	114	99
	Total Fitted/Paid		2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Cal. Per.	42,383		7,151	6,230	4,775	3,690	2,882	2,279	1,807	1,436	1,145	915
Total	41,753		935	979	867	773	693	632	578	526	477	430

1 Unit = $1,000

The results from a forecast scenario can be conveniently summarised in an Accident Year Summary table.

Accident Year Summary Scenario 1:

Accident Yr Summary

Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev.|Data	+-Ult|Data
1997	716	6,391	297	0.41	0.05	177	238
1998	1,008	6,323	376	0.37	0.06	226	301
1999	1,369	7,423	459	0.34	0.06	278	365
2000	1,968	7,410	593	0.30	0.08	364	469
2001	2,671	8,488	730	0.27	0.09	452	573
2002	3,781	9,055	955	0.25	0.11	595	747
2003	5,252	9,794	1,267	0.24	0.13	778	1,001
2004	4,636	7,341	1,168	0.25	0.16	698	936
2005	6,581	7,488	1,619	0.25	0.22	953	1,309
2006	8,103	8,131	2,050	0.25	0.25	1,284	1,599
Total	36,088	77,838	8,514	0.24	0.11	5,030	6,870

1 Unit = $1,000

The two columns on the right describe the changes in SD and Ultimate that can be expected when the next years data has been added. For example, with one more diagonal of the table filled in with observations, there are fewer cells to forecast, so the SD of the aggregate will drop to be in the range of £5,030K. Similarly the Ultimate will undergo some adjustment and the standard deviation of this change is £6,870K.

Accident Year Summary Scenario 2:

Accident Yr Summary

Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev.| Data	+-Ult| Data
1997	814	6,489	320	0.39	0.05	206	244
1998	1,145	6,460	398	0.35	0.06	262	300
1999	1,554	7,608	473	0.30	0.06	318	350
2000	2,232	7,674	592	0.27	0.08	409	428
2001	3,028	8,842	699	0.23	0.08	495	494
2002	4,283	9,557	873	0.20	0.09	627	607
2003	5,950	10,492	1,122	0.19	0.11	783	803
2004	5,259	7,964	1,093	0.21	0.14	776	769
2005	7,467	8,371	1,495	0.20	0.18	1,027	1,086
2006	9,122	9,150	1,836	0.20	0.20	1,344	1,251
Total	40,852	82,605	7,578	0.19	0.09	5,399	5,318

1 Unit = $1,000

Accident Year Summary Scenario 3:

Accident Yr Summary

Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev.| Data	+-Ult| Data
1997	863	6,538	348	0.40	0.05	233	258
1998	1,214	6,529	436	0.36	0.07	299	318
1999	1,648	7,702	525	0.32	0.07	368	374
2000	2,368	7,810	667	0.28	0.09	479	464
2001	3,213	9,027	803	0.25	0.09	590	545
2002	4,546	9,820	1,025	0.23	0.10	762	686
2003	6,313	10,855	1,337	0.21	0.12	970	920
2004	5,575	8,280	1,263	0.23	0.15	933	851
2005	7,913	8,817	1,737	0.22	0.20	1,249	1,208
2006	9,738	9,766	2,187	0.22	0.22	1,647	1,439
Total	43,390	85,143	9,022	0.21	0.11	6,699	6,043

1 Unit = $1,000

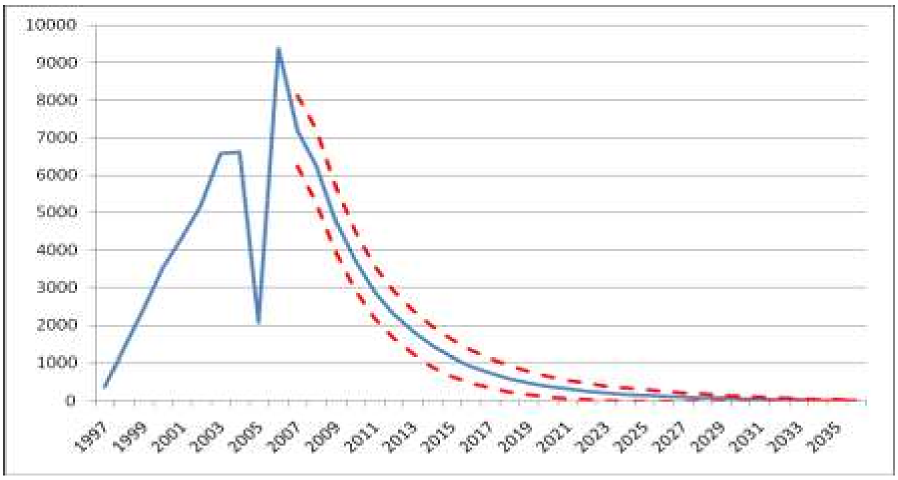
Accident Year Summary Scenario 4:

Accident Yr Summary							
Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev. | Data	+-Ult | Data
1997	929	6,604	386	0.41	0.06	230	309
1998	1,308	6,623	489	0.37	0.07	294	390
1999	1,776	7,830	595	0.34	0.08	361	473
2000	2,554	7,996	770	0.30	0.10	472	608
2001	3,467	9,281	947	0.27	0.10	587	744
2002	4,907	10,181	1,239	0.25	0.12	772	969
2003	6,816	11,358	1,644	0.24	0.14	1,009	1,298
2004	6,016	8,721	1,515	0.25	0.17	906	1,214
2005	8,540	9,444	2,101	0.25	0.22	1,236	1,699
2006	10,515	10,543	2,661	0.25	0.25	1,666	2,075
Total	46,828	88,581	11,049	0.24	0.12	6,527	8,915

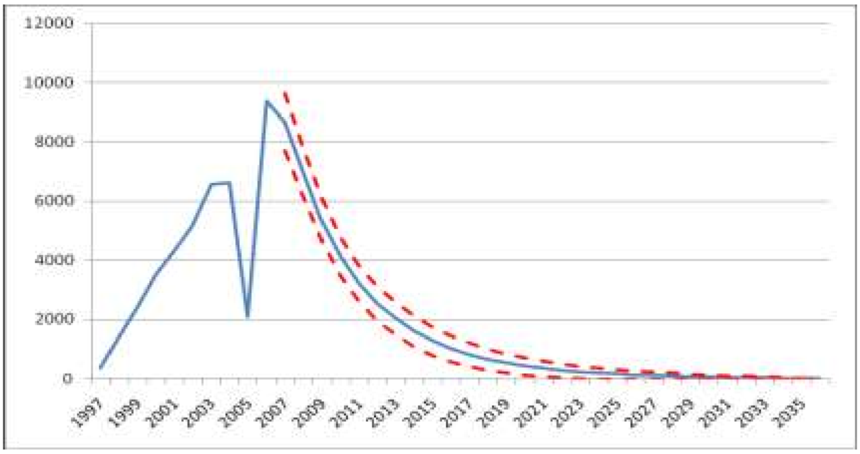
1 Unit = $1,000

Forecast Results – Calendar period payment streams.

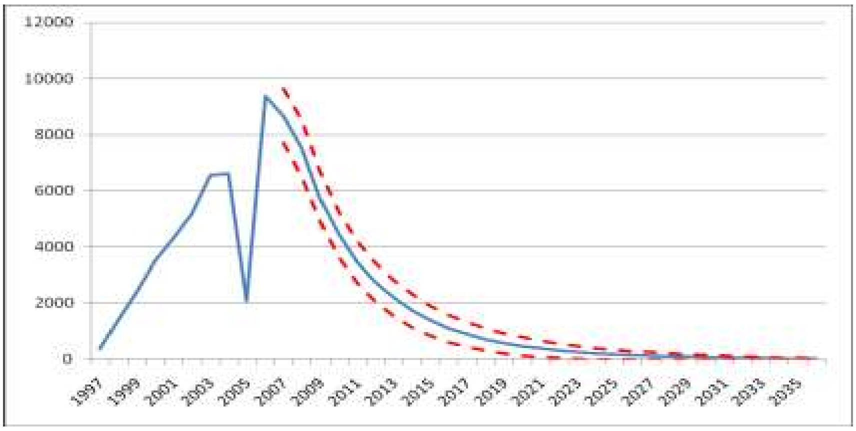
Using the forecast table we can predict the payment streams resulting from the accident years in our data sample. We present these is graphical form below. The red dashed lines begin at the onset of the forecast period and represent the mean +_ one standard deviation.



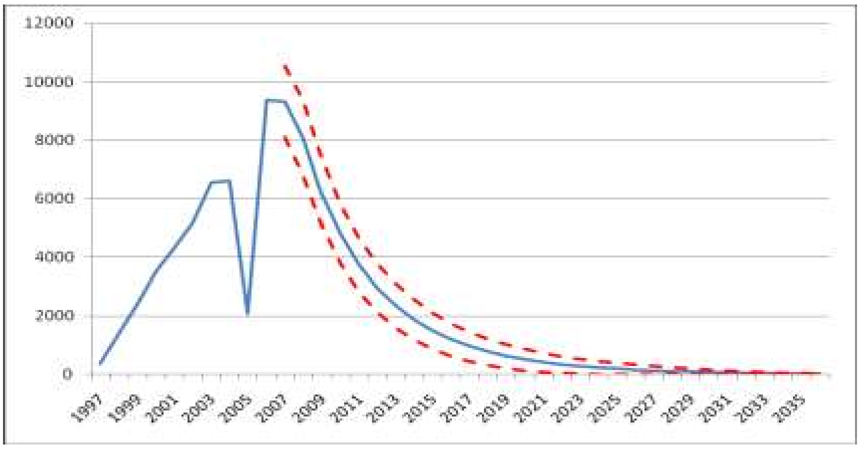
Forecast Scenario 1



Forecast Scenario 2



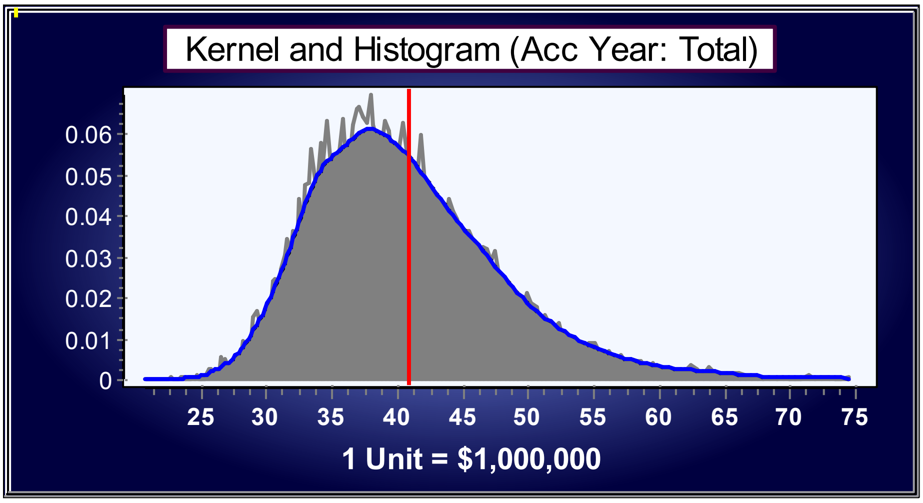
Forecast Scenario 3



Forecast Scenario 4

Reserve probability distributions by calendar year, accident year and total

For brevity we show only the highlights of the reserve probability distribution for Scenario 3.



The histogram of the distribution based on 10000 simulations with Kernel smoothed distribution curve in blue. The red vertical bar is the mean and the yellow the median. The separation between median and mean is an indication of the high degree of skewness.

Quantile Summary for Accident Years

(Sample Distribution)

Accident	Quantiles			
	90%	95%	99%	99.5%
1997	1,205	1,401	1,880	2,077
1998	1,631	1,857	2,454	2,719
1999	2,138	2,406	3,083	3,382
2000	2,967	3,300	4,082	4,536
2001	3,887	4,274	5,190	5,639
2002	5,376	5,818	6,887	7,528
2003	7,327	7,914	9,224	9,945
2004	6,633	7,161	8,355	8,798
2005	9,391	10,122	11,725	12,337
2006	11,485	12,395	14,293	15,303
Total	50,289	54,257	63,580	66,956

1 Unit = $1,000

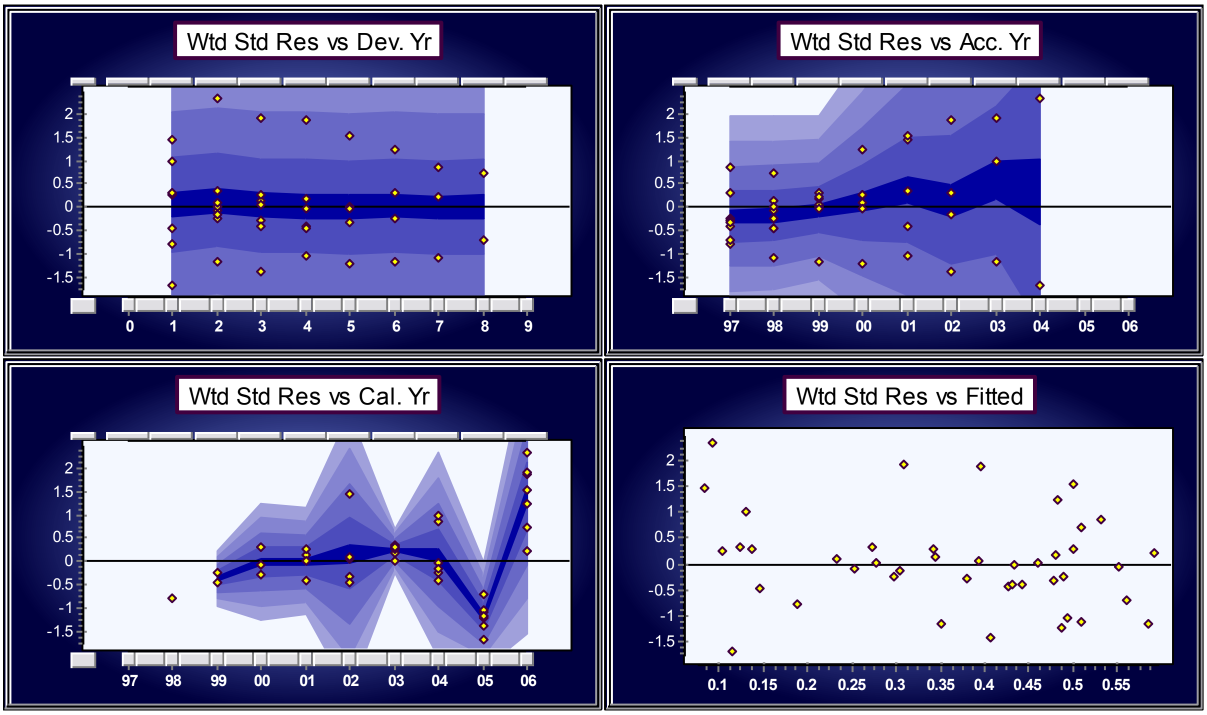
If the Provision is set equal to the Mean the VaR at a given quantile is equal to the distributional value at that quantile minus the mean. The Tail VaR, or Expected Shortfall is a higher number, being the mean value of the losses minus the mean, given that the losses exceed the given quantile.

	Quantiles			
	90%	95%	99%	99.5%
VaR	**9,437**	**13,405**	**22,728**	**26,104**
TVaR	**15,434**	**19,541**	**29,609**	**35,822**

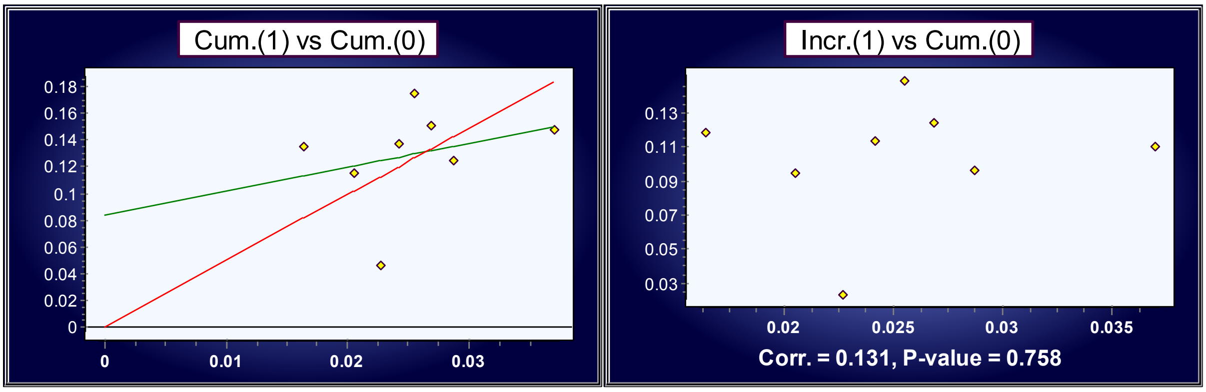
Mack Method PL(C)

We can compare the results above with those produced by the Mack Method as applied to PL(C).

The Mack residual display shows the characteristic “kink” in the calendar direction and indications of an inflationary trend apart from this. There is no way of measuring this however, as the method does not include calendar year parameters a la the PTF modelling framework and does not allow for control of assumptions for the future! There is also a downward trend in Residuals vs. Fitted values.



The predictive power of this method can also be indicated by plotting cumulative for development year n against incremental for year n+1. The results in this case are typically not favourable.



The Mack regression is represented by the red line in the left display. The right display however indicates that the incremental for dev 1 are not predicted well by the cumulative for dev 0. The green line indicates a better fit and illustrates that this method would be improved by an intercept parameter.

Accident Yr Summary

Acc. Yr	Mean Outstanding	Mean Ultimate	Standard Dev.	CV Outstanding	CV Ultimate
1997	0	5,675	0	****	****
1998	329	5,644	145	0.44	0.03
1999	560	6,614	317	0.57	0.05
2000	823	6,265	366	0.44	0.06
2001	1,288	7,102	486	0.38	0.07
2002	1,988	7,262	644	0.32	0.09
2003	3,054	7,596	1,027	0.34	0.14
2004	3,661	6,366	1,436	0.39	0.23
2005	3,387	4,291	2,490	0.74	0.58
2006	652	680	1,215	1.86	1.79
Total	15,742	57,495	3,804	0.24	0.07

1 Unit = $1,000
CV of forecast for last accident yr is 186.38 %
Model may be inappropriate

The Mack reserve mean of 15,742 is very low and far below the identified PTF model mean estimates for all Scenarios 1-4

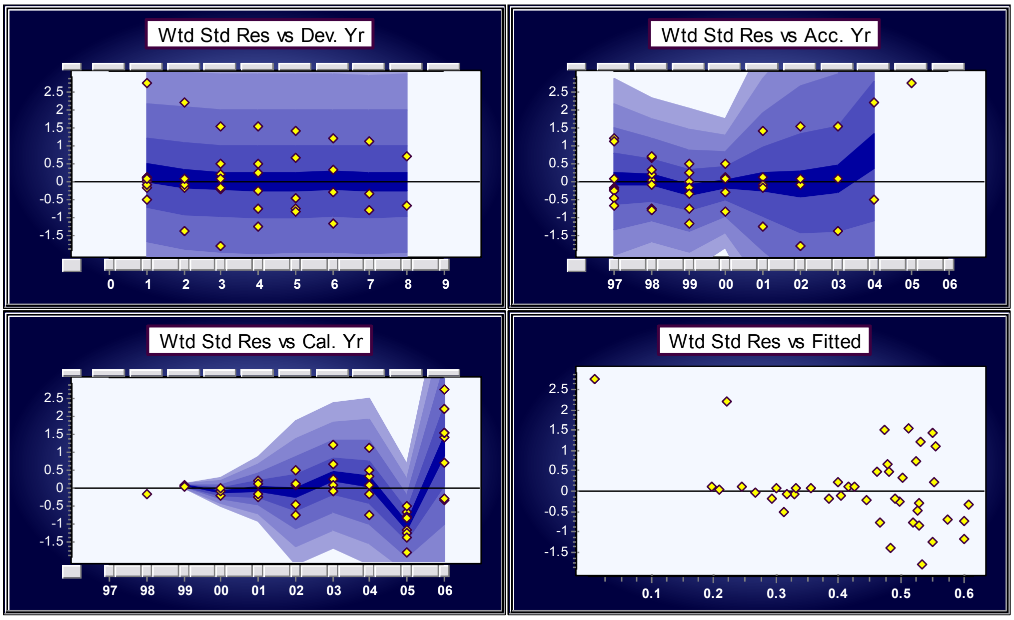
We can obtain almost exactly the same mean and SD as the Mack method if we assume for the identified PTF model the future calendar year trend is -0.25 +_ 0.08 over the same 10 year horizon.

In a similar vein we can capture the mean and SD of the Mack IL(C) forecast in PTF by a scenario that involves a sharp drop in 2006-2007 -0.69+_0.19 followed by a zero trend.

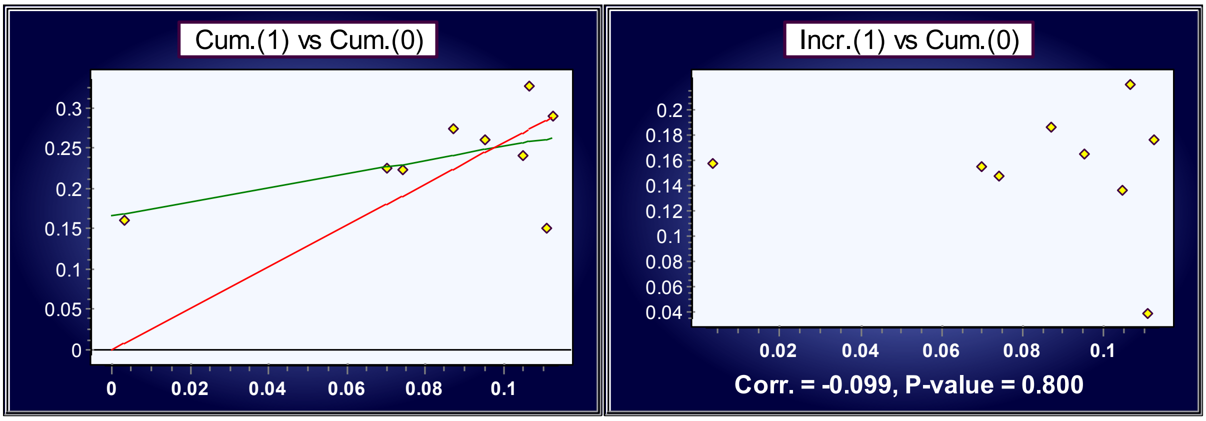
There is, however nothing in the past history of the paid losses to suggest that such forecast scenarios are plausible.

Mack Method IL(C)

The Mack Method applied to the Incurred losses produces similar results to the cumulative paid Losses. If anything the residual display shows more unaccounted-for structure and the Cumulative vs. Next Incremental plots even less correlation.



Mack Residuals for IL(C)



Mack regression (red line, left graph) and the basis for prediction of the innovation (i.e. next incremental) in the right graph.

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Accident Yr Summary

Acc. Yr	Mean		Standard	CV	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate
1997	95	5,770	0	****	****
1998	310	5,625	66	0.21	0.01
1999	494	6,548	165	0.33	0.03
2000	729	6,171	183	0.25	0.03
2001	1,114	6,928	223	0.20	0.03
2002	2,146	7,420	349	0.16	0.05
2003	3,785	8,327	588	0.16	0.07
2004	3,818	6,523	880	0.23	0.13
2005	3,860	4,764	1,273	0.33	0.27
2006	1,362	1,390	3,671	2.70	2.64
Total	17,713	59,466	4,171	0.24	0.07

1 Unit = $1,000
CV of forecast for last accident yr is 269.54 %
Model may be inappropriate

The Mack forecast mean results for the IL(C) are a little higher than for PL(C) but still far below the identified PTF model mean estimates for all Scenarios 1-4.

These mean estimates can be reproduced by the identified PTF model by using the implausible forecast scenario of a negative calendar year trend of -0.2+_0.08 for the remaining 10 years of the forecast period

The Bootstrap, optimal PTF model, and the Mack method

We conclude this section with a discussion of the bootstrap diagnostic tool with application to the optimal PTF model and the Mack method as applied to the Paid Loss data.

The bootstrap is a useful technique for obtaining estimates of parameters and their properties where the sample size is small or where calculation by other methods is intractable. Note that where there are strong calendar trends or other structure in the model, bootstrapped samples are unable to remove the deficiencies in the model and give misleading answers.

Bootstrap samples must come from the same distribution. We have to transform the data into a form where the criteria can be met. To do this, we first create a model for the data that takes the structure out of the data and leaves 'unstructured' (random) noise.

Data = Structure + Noise; or, equivalently: Data = Fitted Model + Residual

The noise can be sampled by the bootstrap without any distributional assumptions. We can sample the noise and add this back onto our model to obtain 'sample data'.

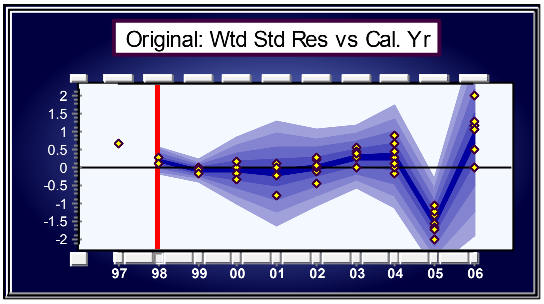
If the model has removed all the structure from the data (the noise component contains no further structure), then the bootstrapped samples should have the same characteristics as the original, real, sample. In this case, a bootstrap sample and the real data are indistinguishable in respect of trend structure and volatility around the trend structure.

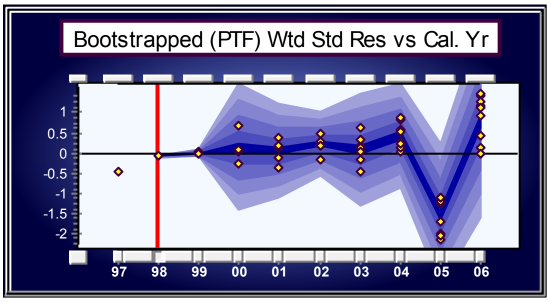
However, if there is still structure existing in the noise component, then the bootstrapped samples will still suffer from the same problem as samples taken from the original data – the calculation of the statistics will not be representative of the underlying population. **The bootstrap technique cannot compensate for a bad model.**

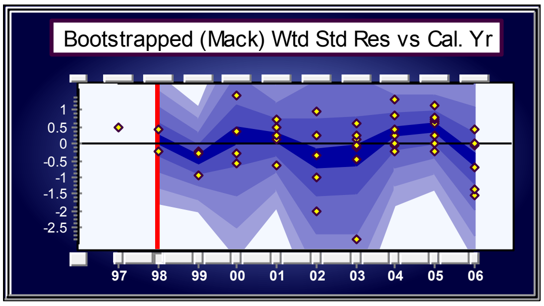
The optimal PTF model and the Mack method are both used to estimate the structure in the data and therefore both produce a set of residuals. The bootstrap can then applied to the sets of residuals with the result being a new sample of pseudo-data. The new samples are expected to be consistent with the original data if the respective fitted model removed all structure from the data.

From our previous analysis, we know that the residuals from the Mack method contain calendar year structure – all the residual points in 2006 are positive while all the residuals in the previous year are negative. Since the Mack residuals are not random, bootstrapping this model will not add any gain in effectiveness in estimating the required reserve. The bootstrapped samples will give significantly different answers from the Mack method applied to the original data and any bootstrapped sample will be instantly distinguishable from the real data.

Residuals from the PTF model are bootstrapped and applied to the fitted PTF model structure to produce new bootstrapped data. The same process is applied to the Mack method. A model which removes the average development and accident level structure was applied to all three datasets – original, bootstrapped from PTF, and bootstrapped from Mack. The structure was then analysed in the calendar direction (residuals versus calendar).

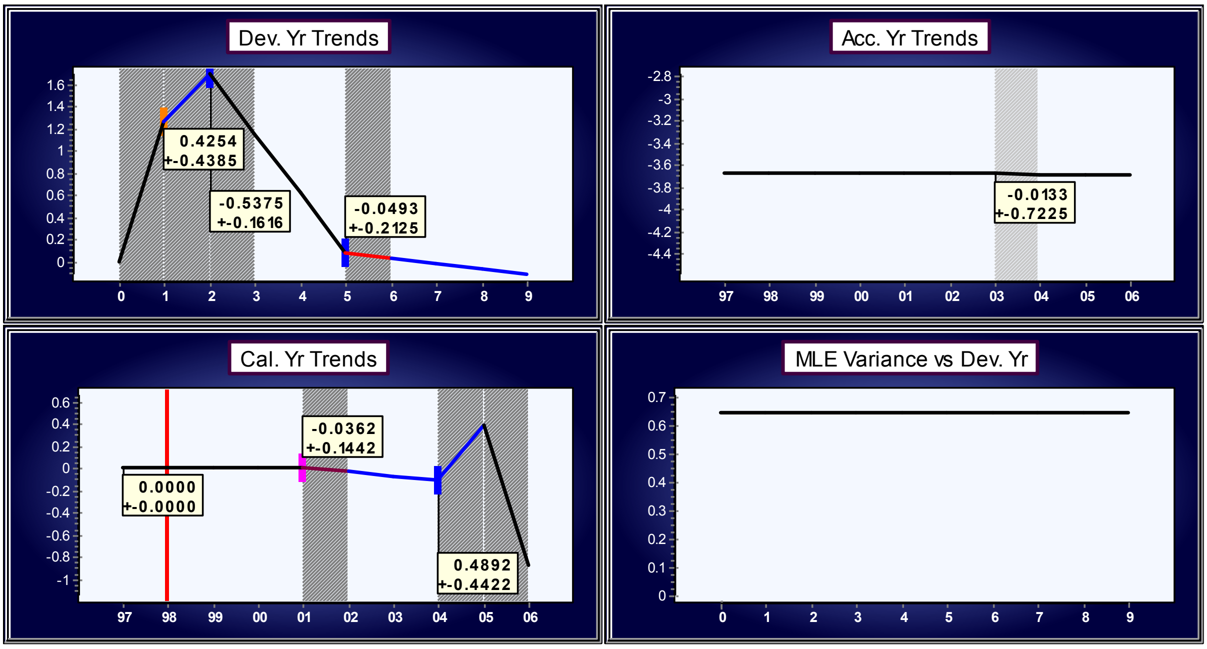




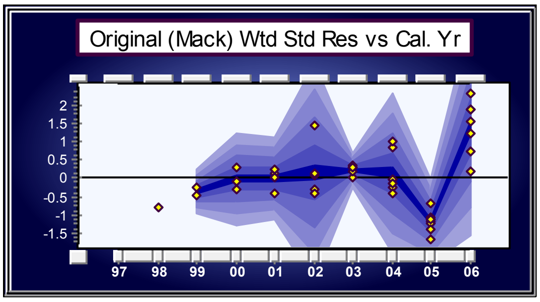


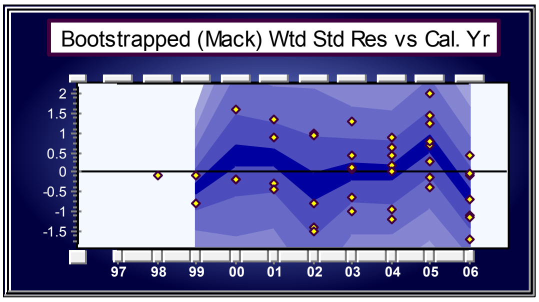
The previous displays highlight instantly that the new data bootstrapped from the Mack method is not comparable to the original data – the calendar year structure has been lost. In contrast, the bootstrapped sample from the PTF model is interchangeable with the original data – the sample produced is comparable to the original data.

The good PTF model, when fitted to the bootstrapped Mack data, gives a completely different picture.



Similarly, applying the Mack method to both datasets, original and the data from the bootstrapped Mack also produce very different results by calendar period.





Again both plots are instantly distinguishable.

Finally, some actuaries bootstrap the Log-linear Poisson model and apply these residuals to the fitted Mack method to obtain new data. There are two problems with this approach. Firstly, the residuals for the log-linear Poisson are very close to the residuals for the chain ladder model for the real data (see previous page) and still have the same structure problem as for the original Mack method. Secondly, there is no reason to expect that the addition: Mack + Bootstrapped residuals (Log-Linear Poisson model) will produce a sample like the original data. In fact, there is every reason to believe that this calculation will not represent the original data since the models are measuring very different structural components.

Conclusion

Applying the bootstrap to the Mack method does not improve the effectiveness of the method at estimating the total reserve rather the deficiencies of the method are shown. Any inference drawn from samples from the Mack method will not represent the real data. As illustrated above, the bootstrapped samples have no connection to the real data since the method did not remove all the structure found in the data.

The bootstrap is a powerful diagnostic tool which further validates the selection of the PTF model.

Data set I

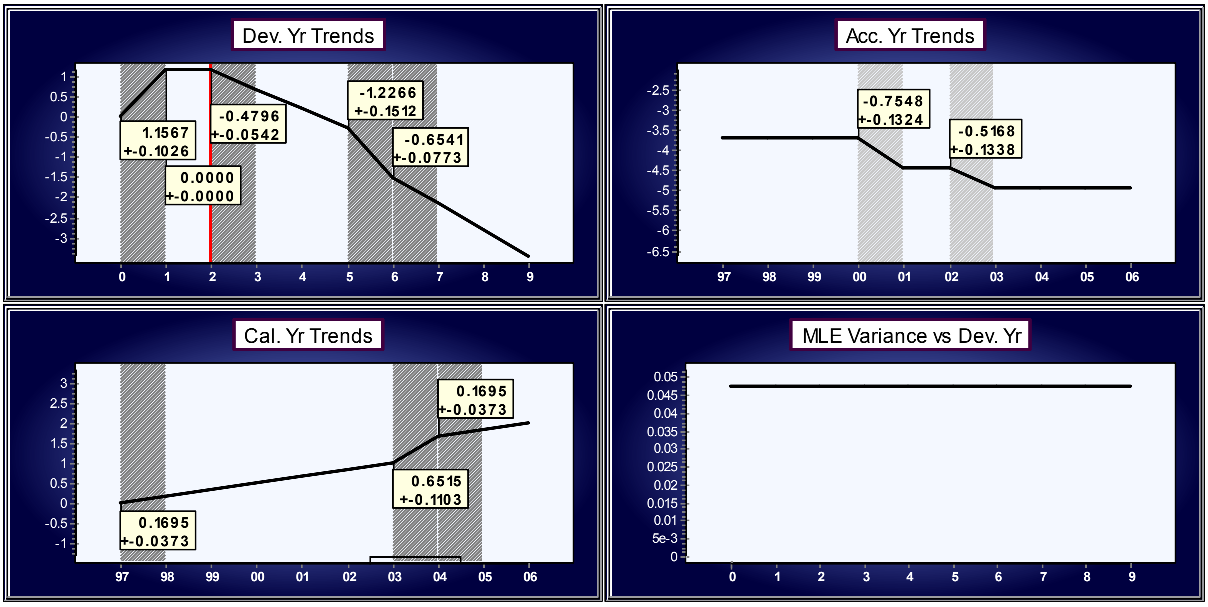
The choice of exposure vector is carried out in the same way as indicated above, so we omit the details. In this case again we chose Wages as the best exposure.

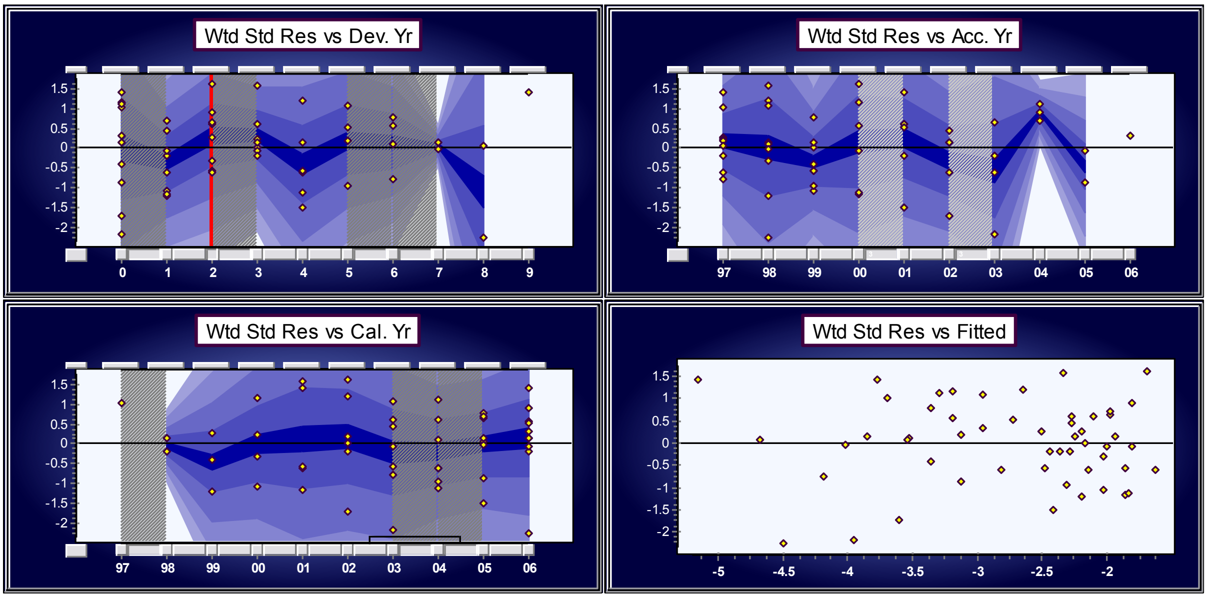
Model Display PL(I)

The PTF Model and Residual Displays appear below.

We note an underlying calendar trend of 0.1695+_0.0373 which is interrupted in 03-04 by a transient occurrence of a much sharper trend 0.6515+_0.1103.

After accounting for this trend change as well as the development and accident structure we obtain residual plots that are substantially flat.

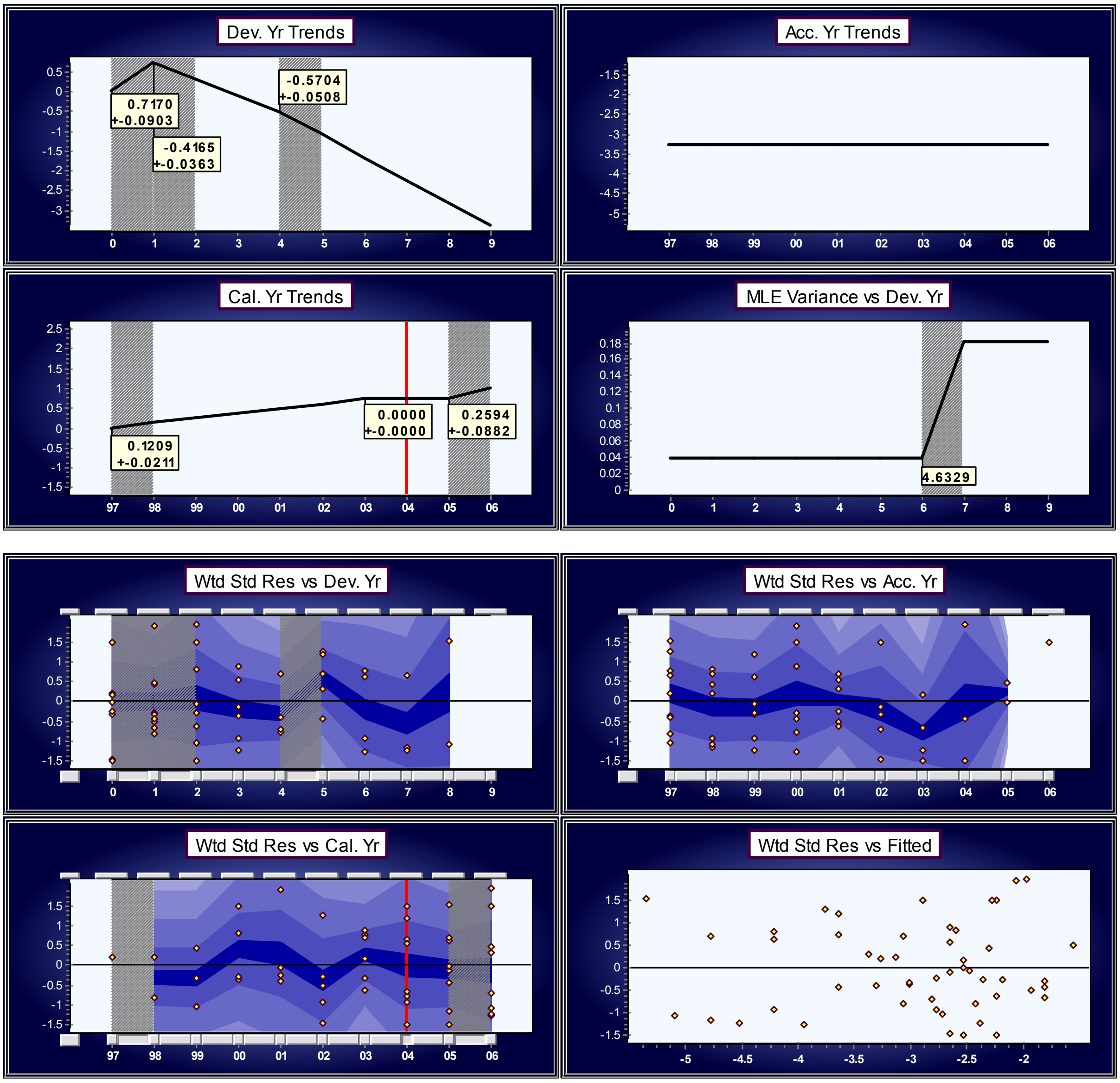




Model Display CRE

The model and Residual displays for the CREs are shown below. Once again, as in the case of dataset H, we see no indication of the same “kink” in the inflationary trend as was seen in the Paid losses, in fact if anything the CREs move in the opposite direction, since the underlying positive trend is suspended in 03-05 and resumes at a sharper rate in 05. One could argue that the more recent large trend in the CREs is catch up.

Once again we conclude that the underlying causes for the increase in PL in 03-04 were not external economic drivers but most likely some change in claim clearing procedures or legislative change. A better understanding of the cause of the change and the modelling of NCC(I) would assist in determining whether this large trend would rear its ugly head again for one year. Lacking the NCC(I) data triangle we cannot carry this inquiry any further



Forecast Scenarios

It may be as for dataset H that substantially more claims are closed in 04 causing the 0.6515+_0.1103 trend. The zero trend in the CREs from 03-05 is suggestive of this but that is hard to know without modelling the NCC.

We present three forecast scenarios in increasing order of conservativity.

Perhaps the most plausible conservative scenario is to continue the underlying 0.1695+_0.0373 trend seen in all years apart from 03-04 for the remainder of the forecast period. We call this Scenario 1.

A second slightly more conservative scenario can be formed by taking the 0.2594+_0.0882 trend seen in the last calendar year of the CREs for the first year of the forecast period for the paid losses before reverting to 0.1695+_0.0373. It is likely that the latest trend in the CREs is catch up as a result of zero from 04-05, but there is some evidence of an increase in the last calendar year of the PL and so we use this figure as an estimated trend in Scenario 2.

A very conservative scenario is that the 0.6515+_0.1103 calendar trend experienced in 03-04 kicks in immediately from 2006 to 2007 and then reverts to 0.1695 +0.0373. This scenario we call Scenario 3.

We would also expect that the negative trend in the development direction would moderate as we go further into the development cycle. We can already see some evidence of this effect, typical of personal injury lines in the shift from the sharp trend D5-D6 to the softer trend D6-D8. Accordingly, we adjust the development trend beyond D8 to -0.5+_0.0773

Forecast Results and Autovalidation with Identified PTF model

The lower left corner of the forecast table for Scenario 1 is shown below. Note the very close fit between the observed and model results for the past portion of the table.

1999	2,627	366	1,378	1,633	1,197	879	1,047	364	223	138	86	63
	2,316	324	1,041	1,387	1,170	750	812	425	225	35	25	22
2000	3,997	446	1,678	1,992	1,460	1,736	1,275	443	272	169	105	77
	3,686	574	1,243	2,548	1,395	1,295	484	491	67	44	32	28
2001	5,160	256	964	1,142	1,355	994	731	255	157	97	60	44
	4,894	349	900	1,287	1,523	679	801	67	40	26	19	16
2002	5,628	304	1,145	2,198	1,609	1,181	868	303	186	115	72	53
	6,508	196	1,235	1,849	1,622	1,188	215	81	48	32	23	20
2003	5,541	224	1,362	1,611	1,181	868	639	223	137	85	53	39
	5,596	129	1,146	1,828	1,097	214	168	62	38	25	18	15
2004	8,893	740	2,788	3,299	2,420	1,779	1,310	457	281	174	109	80
	8,159	938	3,206	3,971	578	441	347	129	79	53	37	32
2005	9,972	1,054	3,967	4,707	3,453	2,539	1,870	653	402	249	156	114
	9,349	831	3,811	1,147	849	649	511	191	118	79	56	48
2006	12,448	1,249	4,699	5,583	4,097	3,012	2,219	775	478	297	188	136
	13,041	1,313	1,145	1,428	1,059	810	635	237	148	100	71	60
	Total Fitted/Paid		2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Cal. Per.	55,764		14,342	12,252	8,681	5,821	3,479	1,582	1,007	659	454	336
Total	55,040		2,017	1,986	1,530	1,150	766	360	266	215	182	163

1 Unit = $1,000

Accident Period Summary for Scenario 1:

Accident Yr Summary							
Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev. | Data	+-Ult | Data
1997	169	4,904	93	0.55	0.02	55	75
1998	277	6,214	129	0.46	0.02	77	103
1999	474	6,608	177	0.37	0.03	106	141
2000	852	9,185	247	0.29	0.03	152	195
2001	744	6,253	158	0.21	0.03	103	120
2002	1,754	7,844	311	0.18	0.04	157	268
2003	2,160	6,360	372	0.17	0.06	222	299
2004	6,850	14,967	1,090	0.16	0.07	702	833
2005	14,458	19,130	2,268	0.16	0.12	1,482	1,717
2006	21,893	23,206	3,559	0.16	0.15	2,644	2,382
Total	49,662	104,702	6,581	0.13	0.06	4,358	4,930
1 Unit = $1,000							

Accident Period Summary for Scenario 2:

Accident Yr Summary							
Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev. | Data	+-Ult | Data
1997	186	4,921	106	0.57	0.02	61	87
1998	306	6,243	148	0.49	0.02	84	122
1999	522	6,656	208	0.40	0.03	116	173
2000	940	9,273	301	0.32	0.03	164	252
2001	817	6,356	190	0.23	0.03	113	152
2002	1,927	8,017	385	0.20	0.05	173	344
2003	2,368	6,568	438	0.18	0.07	245	362
2004	7,521	15,638	1,357	0.18	0.09	780	1,110
2005	15,930	20,572	2,974	0.19	0.14	1,653	2,472
2006	24,105	25,418	4,794	0.20	0.19	2,942	3,785
Total	54,623	109,663	9,220	0.17	0.08	4,857	7,837
1 Unit = $1,000							

Accident Period Summary for Scenario 3:

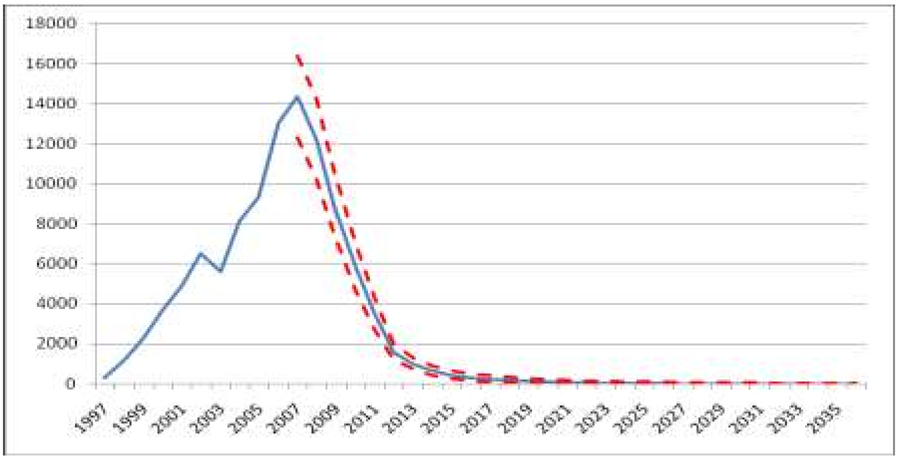
Accident Yr Summary

Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev.|Data	+-Ult|Data
1997	276	5,011	161	0.58	0.03	91	132
1998	454	6,391	226	0.50	0.04	126	187
1999	776	6,910	319	0.41	0.05	172	269
2000	1,397	9,730	469	0.34	0.05	243	401
2001	1,213	6,752	296	0.24	0.04	169	243
2002	2,559	8,949	609	0.21	0.07	257	552
2003	3,511	7,711	682	0.19	0.09	365	576
2004	11,157	19,274	2,157	0.19	0.11	1,161	1,818
2005	23,649	28,291	4,792	0.20	0.17	2,456	4,115
2006	35,808	37,118	7,762	0.22	0.21	4,350	6,429
Total	81,098	136,138	15,164	0.19	0.11	7,184	13,354

1 Unit = $1,000

We show the calendar period payment stream graph only for scenario 1.

Calendar Period Payment Stream Scenario 1:

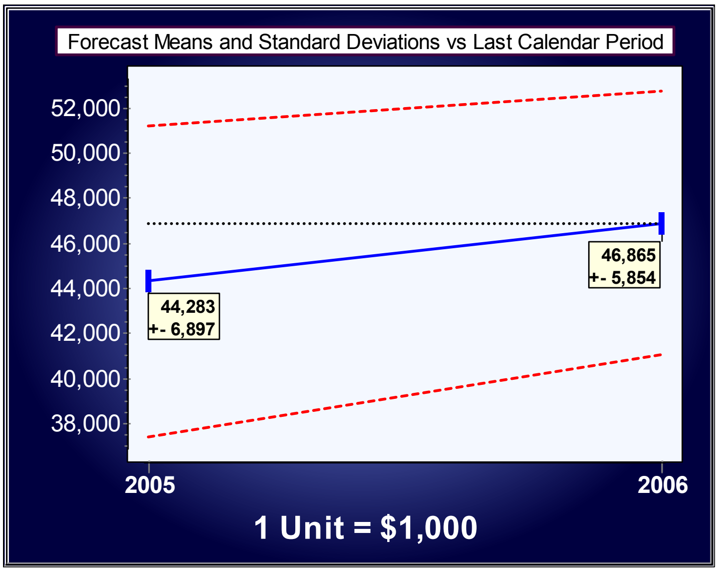


The decay is rapid after the first future year. The stream is 99% exhausted by 2019.

Autovalidation.

If we omit the last diagonal and recalibrate the model to the censored dataset we obtain a slightly lower forecast although well within the predicted range.

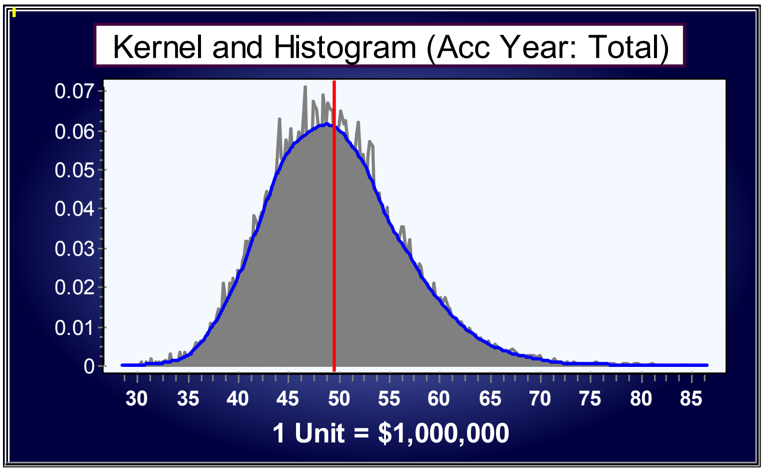
This lends some credibility to our forecast Scenario 2 in which we assume a moderately high inflationary trend in 2006-2007.



Validation comparison for Scenario 1. Note that the means and SDs here refer to completing the forecast square and not to the extended 30 DY forecast used above.

Reserve probability distributions by calendar year, accident year and total

In the interest of brevity we give this only for Scenario 1



Distribution histogram with Kernel smoothed curve. There is some skewness evident but to a smaller degree than was seen for dataset H.

Quantile Summary for Accident Years (Sample Distribution)				
Accident	Quantiles			
	90%	95%	99%	99.5%
1997	274	333	514	624
1998	427	505	739	862
1999	680	791	1,088	1,265
2000	1,144	1,288	1,657	1,893
2001	937	1,019	1,229	1,327
2002	2,167	2,320	2,614	2,723
2003	2,643	2,815	3,187	3,326
2004	8,287	8,796	9,809	10,157
2005	17,456	18,565	20,639	21,432
2006	26,613	28,290	31,666	32,628
Total	58,281	61,258	67,348	69,741

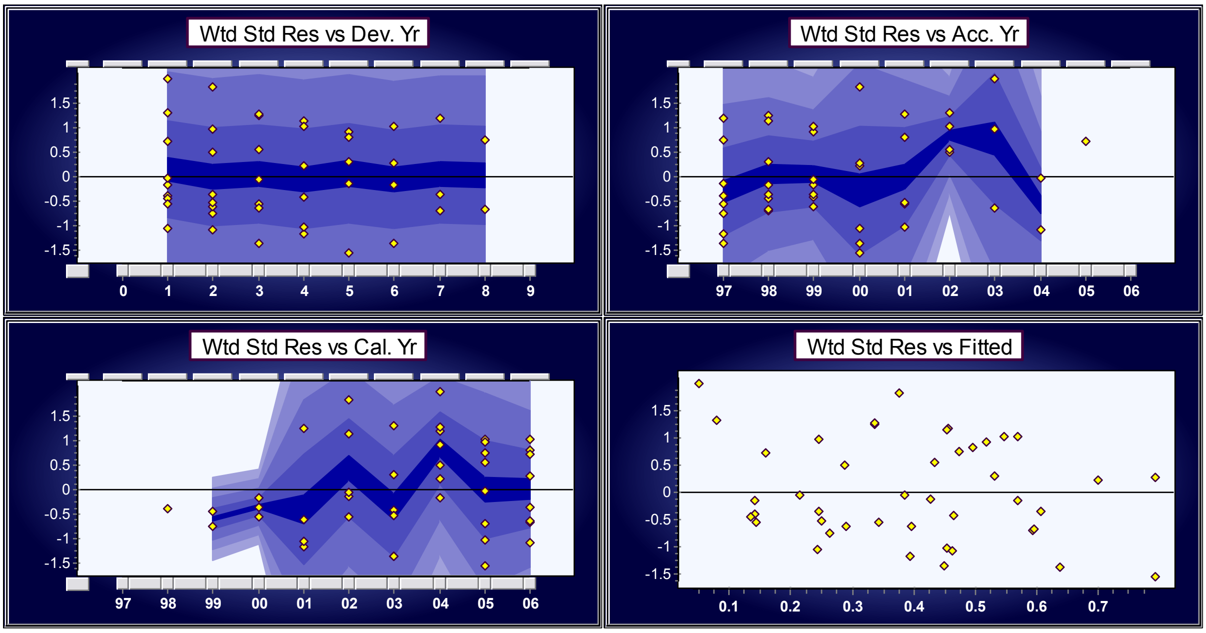
1 Unit = $1,000

	Quantiles			
	90%	95%	99%	99.5%
VaR	8,619	11,596	17,686	20,079
TVaR	12,800	15,576	20,889	22,664

It is notable that the Scenario 2 forecast is at the 80th percentile of the Scenario 1 forecast. Scenario 3 is at the 99th percentile of Scenario 2.

Mack Model and Forecast PL(C)

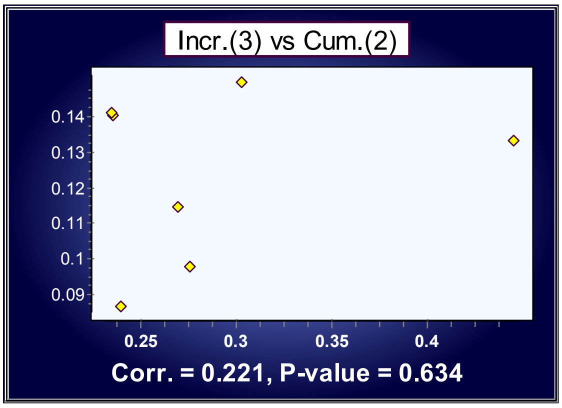
The residual display for the Mack model shows remaining trend patterns in the accident, calendar and fitted directions.



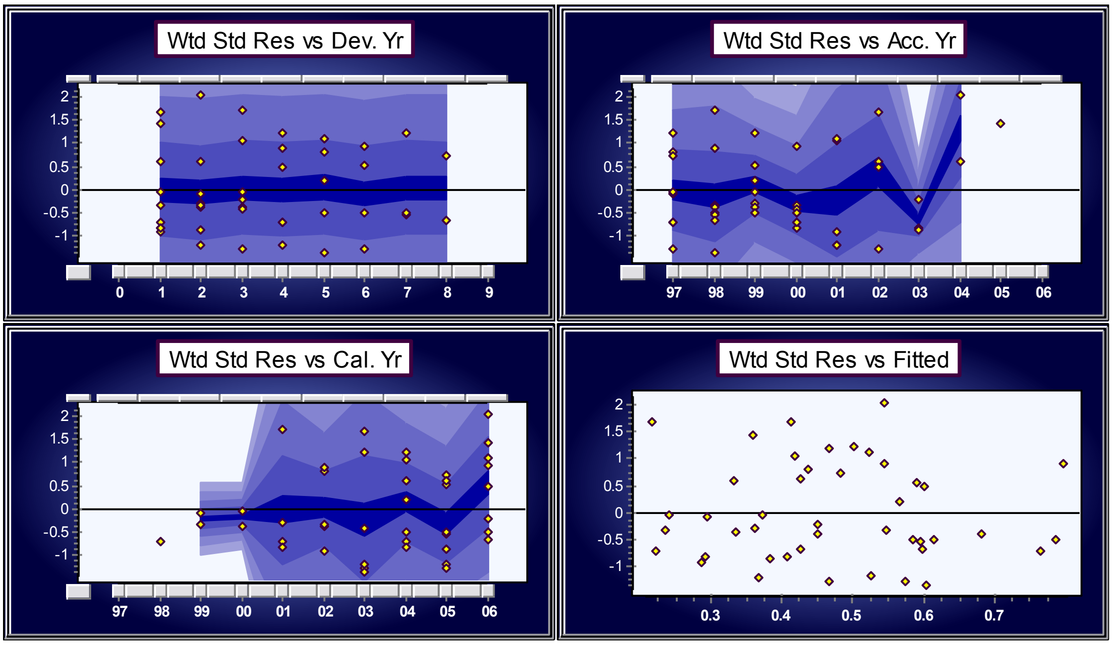
Accident Yr Summary					
Acc. Yr	Mean		Standard	CV	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate
1997	0	4,735	0	****	****
1998	101	6,038	12	0.12	0.00
1999	199	6,333	50	0.25	0.01
2000	652	8,985	178	0.27	0.02
2001	782	6,321	191	0.24	0.03
2002	1,748	7,838	409	0.23	0.05
2003	2,323	6,523	434	0.19	0.07
2004	9,921	18,038	1,702	0.17	0.09
2005	17,732	22,374	3,092	0.17	0.14
2006	26,965	28,278	9,184	0.34	0.32
Total	60,422	115,462	10,314	0.17	0.09
1 Unit = $1,000					

The Mack forecast based on PL(C) comes in higher than those for Scenarios 1 and 2, but lower than Scenario 3, which gives 76,485+_13,600 when limited to the same development range.

The Predictive Power of this model is typically unimpressive:



Mack Model and Forecast IL(C)

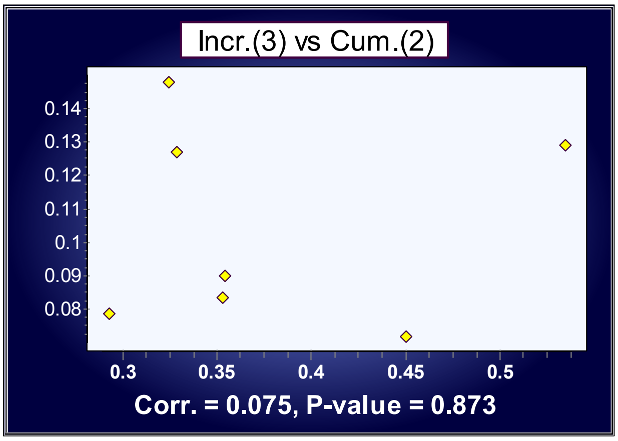


We note a downward trend against fitted and a sharp increase in the last calendar and last two accident periods. Some of this might or might not account for the high forecast figure, which matches the PTF with most conservative Scenario 3.

Accident Yr Summary

Acc. Yr	Mean		Standard	CV	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate
1997	106	4,841	0	****	****
1998	154	6,091	7	0.04	0.00
1999	260	6,394	37	0.14	0.01
2000	672	9,005	174	0.26	0.02
2001	950	6,489	213	0.22	0.03
2002	1,633	7,723	423	0.26	0.05
2003	2,473	6,673	538	0.22	0.08
2004	12,658	20,775	1,914	0.15	0.09
2005	21,249	25,891	3,052	0.14	0.12
2006	35,636	36,949	5,838	0.16	0.16
Total	75,791	130,831	7,771	0.10	0.06

1 Unit = $1,000



A Joint Model for Datasets D, F, H and I.

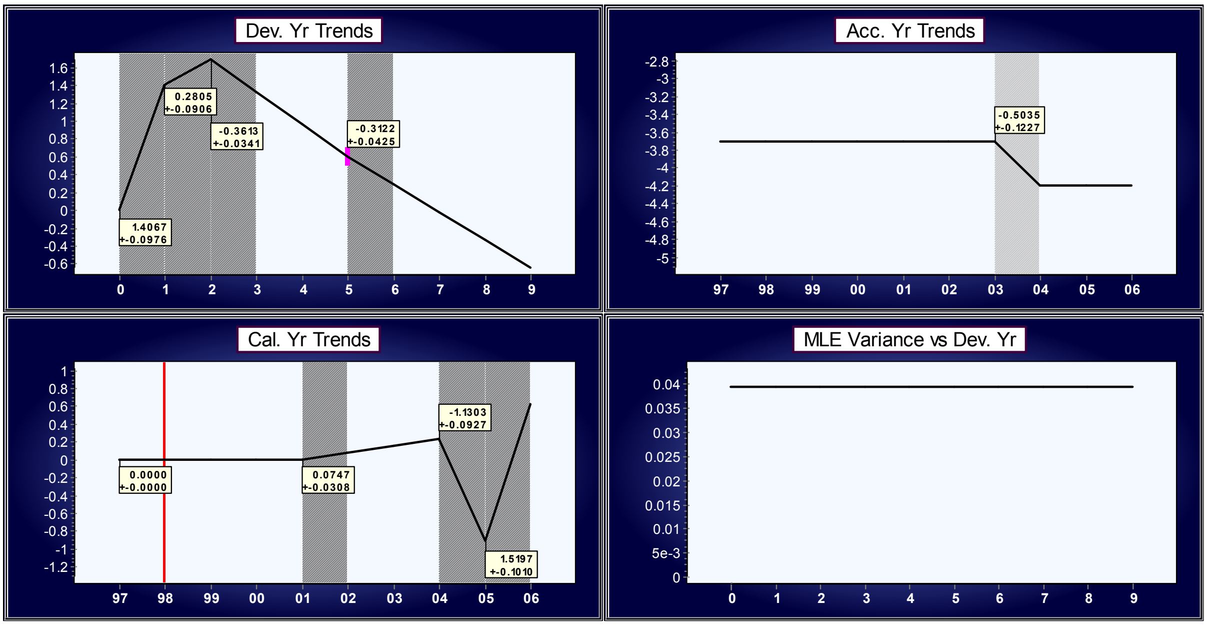
We can combine the four PTF models described above in one joint model in the MPTF framework. This modelling framework detects the process correlations between individual lines and fine-tunes that model parameters using them. The resulting Reserve Correlations are generally smaller than the corresponding process correlations but do have a significant effect on aggregate standard deviations and risk capital allocations. We give the highlights of such an analysis below, under the assumption that **the four datasets correspond to four lines of business in the same company.**

Name	D:PL(I)1	H:PL(I)2	I:PL(I)2	F:PL(I)
D:PL(I)1	1	-0.271447	0.010780	-0.041846
H:PL(I)2	-0.271447	1	-0.054498	0.262407
I:PL(I)2	0.010780	-0.054498	1	-0.191708
F:PL(I)	-0.041846	0.262407	-0.191708	1

The initial process correlation table shows a negative correlation between D and H and a positive correlation between H and F. All of the other correlations are insignificant. We choose to exclude the negative correlation from the final joint model as it may lead to a false optimism. The Final correlation table is as below.

Name	D:PL(I)1	H:PL(I)2	F:PL(I)	I:PL(I)2
D:PL(I)1	1	0.000000	0.000000	0.000000
H:PL(I)2	0.000000	1	0.269220	0.000000
F:PL(I)	0.000000	0.269220	1	0.000000
I:PL(I)2	0.000000	0.000000	0.000000	1

This leads to slight changes in the model parameters for datasets H and F. We illustrate this with the MPTF model display for dataset H as part of the point model, which can be compared with the individual model above in the corresponding section on this dataset.



We retain the same forecast scenarios and for a joint forecast using a scenario based on the preferred scenario in each case but going out to development year 10, so as to be comparable. Comparative plots for the LOBs give an idea of the relative contribution of each line and of the degree of risk diversification enjoyed by their grouping under one umbrella.

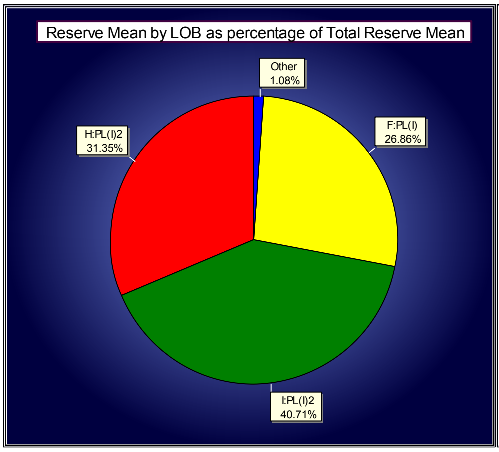
The aggregate forecast table is given below. Note the comparatively low CV. There is clearly considerable diversification benefit from combining the four lines.

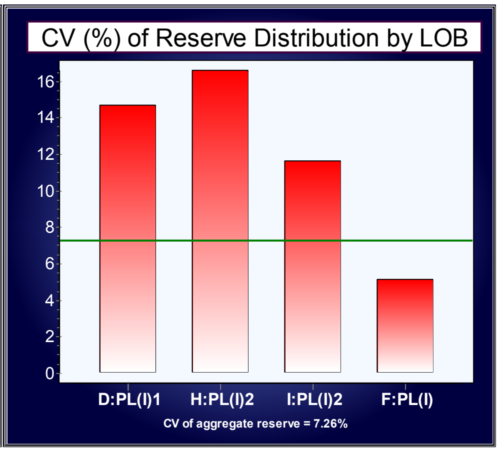
Accident Yr Summary

Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	Std.Dev.|Data	+-Ult|Data
1997	340	17,727	64	0.19	0.00	0	64
1998	568	18,108	127	0.15	0.01	63	111
1999	1,717	21,067	213	0.12	0.01	121	175
2000	3,051	23,277	339	0.11	0.01	203	272
2001	4,738	23,883	516	0.11	0.02	319	406
2002	7,575	25,719	782	0.10	0.03	459	611
2003	11,314	26,105	1,138	0.10	0.04	707	591
2004	17,213	32,755	1,510	0.09	0.05	959	1,166
2005	25,988	37,151	2,646	0.09	0.07	1,710	2,019
2006	40,291	42,214	3,864	0.10	0.09	2,842	2,619
Total	116,394	268,005	5,447	0.07	0.03	5,191	6,663

1 Unit = $1,000

Indeed the Final Reserve Correlation between the positively correlated lines F and H comes to 0.09, so the correlation between H and F does not greatly reduce the diversification.

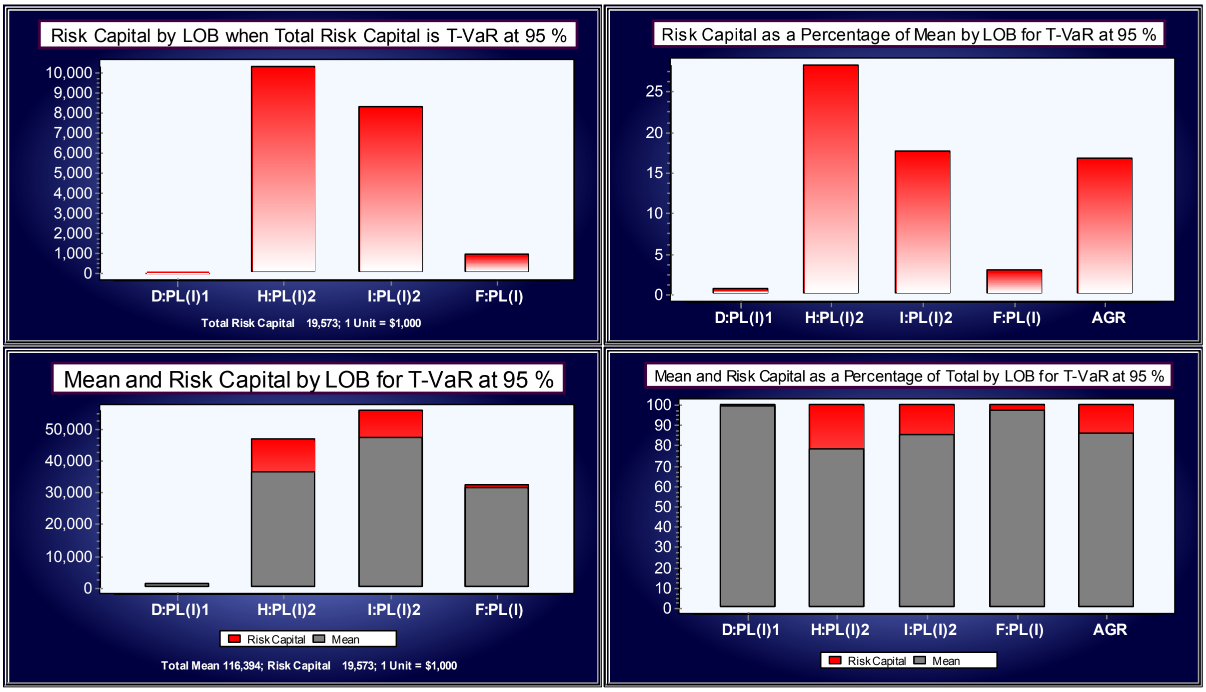




The pie chart on the left shows the comparative size of the LOBs in terms of the mean, and the bar graph on the left the comparison of the CVs. We see that only line F has a CV below that of the aggregate. Line D is very small, accounting for about 1% of the total, while I is the largest line at around 40% of the total.

The graphs below show the comparative Risk Capital Allocation for the four lines, based on a total risk capital equal to the TVaR at the 95th percentile of the aggregate distribution. Capital is allocated to each line in proportion to its contribution to the overall volatility.

The display on the right indicates that H is the riskiest line since it's share of the risk capital runs to more than 25% of its reserve. Increasing the share of business accounted for by D or F should lower the overall risk level of the combined business, which currently is very close to that experienced by I, with risk capital at around 15% of the reserve.



The quantile summary for the aggregate loss distribution shows again that volatility in the aggregate has been significantly contained.

Quantile Summary for Accident Years (Sample Distribution)				
Accident	Quantiles			
	60%	75%	95%	99%
1997	349	379	456	525
1998	891	947	1,091	1,218
1999	1,758	1,853	2,092	2,268
2000	3,113	3,267	3,652	3,967
2001	4,833	5,062	5,641	6,112
2002	8,029	8,370	9,240	9,944
2003	11,523	12,032	13,302	14,372
2004	17,512	18,166	19,836	21,137
2005	29,537	30,692	33,563	35,780
2006	41,067	42,792	47,095	50,629
Total	118,168	121,922	130,955	138,492
1 Unit = $1,000				

Conclusion

The PTF modelling framework is seen to be vastly superior to the alternative link ratio based methods including Mack (volume weighted average link ratios) for many reasons. The identified PTF model captures (describes) the volatility in a loss development array in a succinct way - the loss development array is regarded as a sample (path) from the fitted distributions to each cell. Trend relationships between paid losses, CREs and NCC arrays can be identified and used in formulating assumptions about future trends in the paid losses, especially in the presence of calendar year trend instability in the paid losses. The actuary has control on future assumptions that are explicit, can be related to past volatility, are audit able and can be monitored in a sound probabilistic framework. The numerous benefits include: statistically consistent estimates of prior year ultimates on updating, probability distributions of liability streams by calendar year required for cost of capital calculations, pricing future underwriting (accident) years, and computing the combined reserve and risk (capital) charge.